1次元交通流におけるエントロピーの定式化と相互情報量を利用したメタ安定状態の検知

三浦紋子 [1], 友枝明保 [2], 西成活裕 [3]

[1] データストラテジー株式会社
[2] 武蔵野大学 工学部 数理工学科 [3] 東京大学 先端科学技術研究センター

概要

エントロピーはニュートン粒子の状態をマクロ・ミクロ両面から表現できる状態量である。しかし、ニュートンの法則を満たさない自己駆動粒子について、エントロピーを定義した研究は少ない。本研究では、1次元交通流のエントロピーとその一部である相互情報量を情報理論の観点から再定義した。新しく定義されたエントロピーの実用性を示すため、高速道路を追従走行する3台の車両のGPSデータを用いて、エントロピーの数値計算を行い、相互情報量がメタ安定状態の予想に利用できることを示した。

Formularization of entropy and detection of metastable states using mutual information in one-dimensional traffic flow

Ayako Miura [1], Akiyasu Tomoeda [2], Katsuhiro Nishinari [3]

[1] Data Strategy Inc.
[2] Department of Mathematical Engineering, Faculty of Engineering, Musashino University
[3] Research Center for Advanced Science and Technology, The University of Tokyo

Abstract

Entropy is a useful property that can express both macro and micro states of Newtonian particles. However, in previous research studies, researchers did not fully determine how entropy can be defined for self-driven particles that do not satisfy Newton's laws. In this study, we redefine the entropy and mutual information in one-dimensional traffic flow from the information theoretical perspective. Moreover, we used the mutual information to detect a metastable state by calculating GPS data from three following vehicles on a real expressway to demonstrate the practical application of the formularized mutual information.

1 Introduction

Entropy is defined both from a macro perspective, such as thermodynamics and fluid dynamics, and a micro perspective, such as statistical mechanics and information theory. Therefore, entropy is a useful property that can express both the macro and micro states of Newtonian particles. However, the self-driven particles do not satisfy Newton's first and third laws of motion. In contrast to the usefulness of entropy to define both the macro and micro states of Newtonian particles, researchers are yet to fully determine how entropy can be defined for the non-Newtonian self-driven particles. For instance, Kerner & Konhauser [1] and Reiss et al. [2] performed thermodynamic studies of vehicles that are typical examples of self-

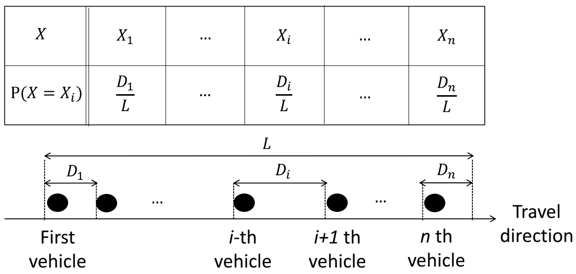

Fig.1: The schematic diagram of the relation between X, $P(X = X_i)$, D_i, L, and vehicles

driven particles [3]. Kerner & Konhauser defined the traffic temperature and pressure in terms of the correspondence between their hydrodynamic models and the Navier–Stokes equation. In addition, Reiss et al. [2] performed further study on one-dimensional traffic flow using thermodynamics to define traffic temperature and pressure. They mentioned entropy but considered that it was not an appropriate property to represent the uncertainty in traffic flows. Therefore, explaining the entropy of vehicular traffic, which is described both by macro- and micro-scale models [3], is a notable challenge.

In this study, we begin with Iwasaki and Sadakata's definition of entropy [4].

$$S(X) = -\sum_{i=1}^{n} P(X = X_i) \log_2 P(X = X_i), \quad (1)$$

where n is the number of vehicles, X is a discrete random variable, X_i is the i-th possible value of X, and $P(X = X_i)$ is a probability distribution function. Note that X_i corresponds to the situation when the headway of the i-th vehicle becomes D_i and $P(X = X_i) = \frac{D_i}{L}$, where L is the length of the road. The schematic of the relation between X, $P(X = X_i)$, D_i, L, and vehicles is shown in Fig. 1. The definition in Eq. (1) is analogous to the formula of entropy commonly used in information theory with the assumption that the headway between the vehicles is a mutually isolated, independent event. Entropy formularized as $S(X)$ represents the variation in the locations of the vehicles. $S(X)$ is maximum when the headway consists of equally spaced intervals and the vehicles are widely scattered on the road, while $S(X)$ is minimum when all the vehicles form a single cluster with the minimum intervals. However, they did not examine the velocity variation of vehicles in traffic flow. In this study, we advance the previous research, which focused on traffic flow from information theory, by defining the new entropy in traffic flow.

2 A New Formulation of Entropy in 1D Traffic Flow

We discretize a random variable Y for the velocity, and we assume the possible values of Y into three values Y_1, Y_2, and Y_3 corresponding to situations when the velocity is 0, between 0 and maximum, and maximum, respectively. Although it is possible to discretize Y in further detail because measured data show that the velocity changes continuously, we categorized Y into three values for the sake of simplicity.

For a fixed length of the road, the joint distribution $P(X = X_i, Y = Y_j)$ can be formularized between the random variables X and Y as listed in Tables 1 and 2. (X_i, Y_1) occurred when the headway of i-th vehicle was D_i and the velocity of vehicle i was 0. This situation reflected that the headway of the vehicle was $D_v + D_s$ in terms of the length of the vehicle D_v and the inter-vehicle gap D_s as the vehicle came to rest. (X_i, Y_2) occurred when the headway of i-th vehicle was D_i and the velocity of vehicle i was between 0 and maximum. It indicated that the headway was longer than $D_v + D_s$ but shorter than $D_v + D_s + D_a$, where D_a represents the threshold value of the inter-vehicle gap to distinguish the maximum velocity from any lower velocity. (X_i, Y_3) occurred when the headway of the i-th vehicle was D_i and the velocity of vehicle i was at maximum; this signified that the headway was longer than or equal to $D_v + D_s + D_a$. A schematic of the relation between Y_j, $P(X_i, Y_j)$, D_v, D_s, D_a, D_i, L, and vehicles is shown in Fig. 2.

$P(X_i, Y_j)$ can be classified based on the magnitude correlation between D_i and $D_v + D_s + D_a$, and these are listed in Tables 1 and 2. Furthermore, when $D_v < D_i \leq D_v + D_s$, $P(X_i, Y_j)$ is the

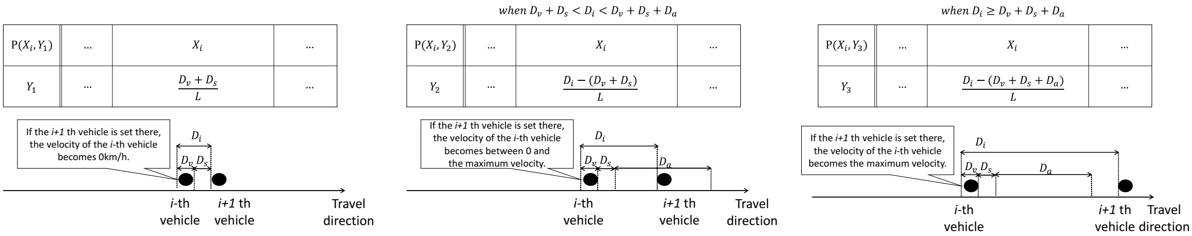

Fig.2: A schematic diagram of the relation between Y_j, $P(X_i, Y_j)$, D_v, D_s, D_a, D_i, L, and vehicles.

marginal distribution of X_i. This formulation is dependent on the assumption that vehicles have a relation between the headway and velocity.

Subsequently, we can formularize entropy in a system of n vehicles. The entropy $S(X)$ representing the location variation is given by Eq. (1), and the entropy $S(Y)$ is defined by

$$S(Y) = -\sum_{j=1}^{3} P(Y = Y_j) \log_2 P(Y = Y_j), \qquad (2)$$

where Y denotes the discrete random variable, Y_j is the j-th possible value of Y, and $P(Y = Y_j)$ is a probability distribution function, as defined in Tables 1 and 2. $S(Y)$ represents the velocity variation of all the vehicles. We assume that the velocities of the individual vehicles are mutually isolated because each vehicle is a self-driven particle. The entropy $S(X, Y)$ considering the location and velocity variations is defined as

$$S(X,Y) = -\sum_{i=1}^{n}\sum_{j=1}^{3} P(X{=}X_i, Y{=}Y_j) \log_2 P(X{=}X_i, Y{=}Y_j),$$

where $P(X = X_i, Y = Y_j)$ is a joint distribution function, as defined in Tables 1 and 2. In accordance with the formalism of information theory, $\frac{0}{L} \log_2 \frac{0}{L} = 0$. $S(X)$, $S(Y)$, and $S(X, Y)$ were calculated when the length of the road L was constant. The three types of entropy described above can be used to derive the mutual information $I(X; Y)$;

$$I(X;Y) = S(X) + S(Y) - S(X,Y). \qquad (3)$$

3 Detection of Metastable States Using $I(X; Y)$

We use the GPS data obtained when three vehicles were platooning on a real expressway in order to verify the characteristics of mutual information. The velocity and headway are calculated from the GPS data. Their average values for the three vehicles are shown in Fig. 3. After 200 s, there is a period during which the average velocity increased; although the average headway values were high in the early half of this period (200 to 400 s), they were small in the latter half (400 to 600 s).

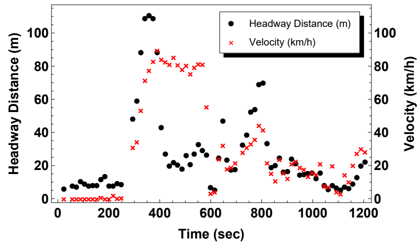

Fig.3: Time evolution of the average velocity values for three vehicles and average headway values between the three vehicles.

We obtained the real data of the following three vehicles in open system. To calculate each entropy defined for a system with periodic boundaries, we assume that the front vehicle varied the speed and the succeeding two vehicles increased or reduced their speed owing to the front vehicle in the circuit $L = 300$ [m]. The circuit length L is set such that it does not exceed the sum of the headways of the succeeding two vehicles. $I(X; Y)$, as shown in Fig. 4, increases when the average velocity is low and headway values are small in congested flow but decreases when the congestion starts to clear. $I(X; Y)$ obtains different values at 200 to 400 s and 400 to 600 s because $I(X; Y)$ varies between a state of high velocity and long headway and in a state of high velocity but short headway. In other words, the mutual information $I(X; Y)$ enables the

Table 1: $P(X_i, Y_j)$ when $D_v + D_s < D_i < D_v + D_s + D_a$.

$P(X_i, Y_j)$	X_1	$\cdots$	X_i	$\cdots$	X_n	$P(Y_j)$
Y_1	$\frac{D_v+D_s}{L}$	$\cdots$	$\frac{D_v+D_s}{L}$	$\cdots$	$\frac{D_v+D_s}{L}$	$\frac{n(D_v+D_s)}{L}$
Y_2	$\frac{D_1-(D_v+D_s)}{L}$	$\cdots$	$\frac{D_i-(D_v+D_s)}{L}$	$\cdots$	$\frac{D_n-(D_v+D_s)}{L}$	$\sum_{i=1}^{n} \frac{D_i-(D_v+D_s)}{L}$
Y_3	0	$\cdots$	0	$\cdots$	0	0
$P(X_i)$	$\frac{D_1}{L}$	$\cdots$	$\frac{D_i}{L}$	$\cdots$	$\frac{D_n}{L}$	1

Table 2: $P(X_i, Y_j)$ when $D_i \geq D_v + D_s + D_a$.

$P(X_i, Y_j)$	X_1	$\cdots$	X_i	$\cdots$	X_n	$P(Y_j)$
Y_1	$\frac{D_v+D_s}{L}$	$\cdots$	$\frac{D_v+D_s}{L}$	$\cdots$	$\frac{D_v+D_s}{L}$	$\frac{n(D_v+D_s)}{L}$
Y_2	$\frac{D_a}{L}$	$\cdots$	$\frac{D_a}{L}$	$\cdots$	$\frac{D_a}{L}$	$\frac{nD_a}{L}$
Y_3	$\frac{D_1-(D_v+D_s+D_a)}{L}$	$\cdots$	$\frac{D_i-(D_v+D_s+D_a)}{L}$	$\cdots$	$\frac{D_n-(D_v+D_s+D_a)}{L}$	$\sum_{i=1}^{n} \frac{D_i-(D_v+D_s+D_a)}{L}$
$P(X_i)$	$\frac{D_1}{L}$	$\cdots$	$\frac{D_i}{L}$	$\cdots$	$\frac{D_n}{L}$	1

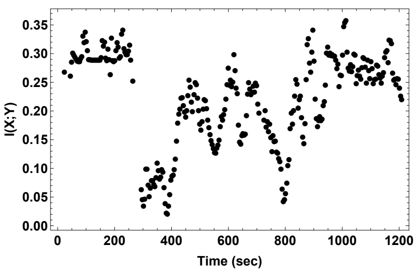

Fig.4: Time evolution of $I(X;Y)$ of three vehicles (calculated with $D_v = 5$ [m], $D_s = 3$ [m], $D_a = 10$ [m]).

detection of a metastable state of high velocity but short headway. The advantages of using I(X;Y) for metastable state detection is that using one parameter for the detection is easier than two parameters, considering the computation of autonomous vehicles.

4 Conclusion

We modified the formularization of entropy in one-dimensional traffic flow from a micro viewpoint. This was proposed in previous research [4] that formularized entropy focusing on headway between vehicles. We redefined entropy focusing on the velocity variation of vehicles to formularize entropy and mutual information. Moreover, when we calculated $I(X;Y)$ using experimental data obtained from three vehicles on a real expressway, we found that although the average velocity of the three vehicles was high, $I(X;Y)$ varied with the average headway between the three vehicles. The detection of a metastable state is an application of $I(X;Y)$ in this study, but it also has a practical application for traffic management as the detection of these metastable states can be a sign of imminent congestion. For further details of this study, please refer to our published article https://doi.org/10.1016/j.physa.2020.125152.

References

[1] B. S. Kerner and P. Konhauser, Phys. Rev. E **48**, R2335(R) (1993).

[2] H. Reiss, A. D. Hammerrich, and E. W. Montroll, J. Stat. Phys. **42**, 3, p.647 (1986).

[3] D. Chowdhury, L.Santen, and A. Schadschneider, Phys. Reports **329**, p.199 (2000).

[4] Y. Iwasaki and M. Sadakata, in *Proceedings of IFORS International Conference on Operational Research, Athens, 1990*, (Elsevier, New York, 1990), p.379.