The effect of inflow rate and conflict around the exit on evacuation efficiency

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Abstract

In this paper, a cluster approximation method based on the floor field model is proposed to analyze the effect of inflow rate and conflicts around the exit during an evacuation process. The conflicts have been taken into account by 2 different approaches: the friction parameter and the friction function. It has been observed that despite the difference of conflict level, there always exists an optimal inflow rate. The calculation results could be considered as the verification of the 'faster is slower' effect. Besides theoretical analysis, we also conducted simulation to validate our model. We found that through adjusting the inflow rate to appropriate value, the evacuation time in all the conflict regime could be approximated with satisfactory accuracy.

1 Introduction

Pedestrian evacuation is vigorously researched over the last decades. In addition to a lot of evacuation experiments conducted to investigate complex behaviors of pedestrians during evacuation process from a perspective of both psychology and physiology [1] [2], various kinds of microscopic models, which enable computer simulation based studies, are developed. Floor field cellular-automata model is a famous decision-based model, it is based on cells in 2 dimensional space and discrete in space, time and the state variables [3]. Despite its simplicity, floor field model could reproduce various complicated evacuation dynamics, thus it is well studied and extended in recent years.

An important phenomenon in evacuation is the so called 'faster-is-slower' effect, which refers to the phenomenon that as individual pushes harder to escape through an exit, an increasement of the total evacuation time can be achieved. Besides the experimental verification, researchers also incorporated the friction parameter [4] and the friction function [5] to reproduce such phenomenon via floor field model simulation. The friction parameter stands for the clogging and sticking effects between evacuees, it describes the probability an evacuee could not proceed to its desired position when engaged in a conflict with other evacuees. Friction function accounts for the fact that the larger the number of evacuees engaged in a conflict, the stronger the competition will be, it treats the probability that all the movements are denied as function of the number of evacuees.

In this research, we investigated the evacuation dynamics near the exit through theoretical analysis and simulation. Because of the complexity of 2 dimensionality, unlike vehicle traffic which has been well theoretically studied by extending the 1 dimensional asymmetric simple exclusion process, only in a very limited number of evacuation researches theoretical analysis has been conducted.

This paper is organized as follows. In Sec. 2, we briefly review the floor field model along with the friction parameter and the friction function. Subsequently we propose the cluster approximation method in Sec. 3, and compare the simulation results with the theoretical calculation. Finally, Sec. 4 is devoted to summary and discussion.

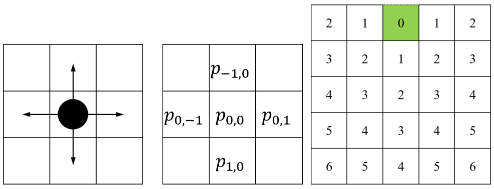
2 Floor Field Model and conflict resolution

2.1 Floor Field

As the focus of this research is the evacuation dynamics near the exit, we consider a square room with only 1 exit. The room is divided into 2 dimensional cells, for each cell, only 2 conditions are available: empty or occupied by an evacuee. At each time step, an evacuee could choose to proceed to a Von Neumann neighbor cell, or stay in its current place (Fig. 1). The decision-making of evacuees is determined by 2 floor fields: the static floor field and the dynamic floor field. The static floor field describes the shortest distance from the exit, as shown in Fig. 1. The dynamic floor field is a bionics-inspired approach, it could be considered as the footprints left by pedestrians, and reflects that under emergency, human-beings are likely to lose their minds and just follow the footsteps of others. By adopting dynamic floor field, the long-ranged interaction among evacuees is mimicked to local-ranged one. The probability an evacuee chooses the cell (i, j) to move into is determined by the following equations.

$$p_{ij} = N\xi_{ij}(1 - n_{ij})\exp(-k_s S_{ij} + k_d D_{ij}), \quad (1)$$

$$N = \left[\sum_{(i,j)} \xi_{ij}(1 - n_{ij})\exp(-k_s S_{ij} + k_d D_{ij})\right]^{-1} \quad (2)$$



(a) (b)

Fig.1: (a) Target cells for an agent, only the Von Neumann neighbor cells are available. (b) Static floor field of a 5×5 room. The exit cell is marked as green. The number in each cell represents the Manhattan distance from the exit cell.

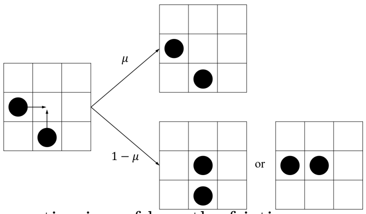


Fig.2: A schematic view of how the friction parameter resolves conflict. The movements of two pedestrians are denied with probability μ, otherwise a pedestrian will be randomly selected and able to proceed.

$$\xi_{ij} = \begin{cases} 0, & \text{for obstacle or wall cells} \\ 1, & \text{otherwise} \end{cases} \tag{3}$$

Here, k_s and k_d are sensitive parameters of the static floor field and the dynamic floor field, respectively. $n_{ij} \in 0, 1$ refers to the occupation number. As in this research we only consider the evacuation from a simple room, we suggest that all the evacuees have a good knowledge of the location of the exit, thus the effect of the dynamic floor field is ignored. For simplicity we set k_s large enough to ensure the effect of static floor field dominates, i.e., $k_s = 10, k_d = 0$, which corresponds to the ordered regime in Ref. [3].

2.2 Friction Parameter and Friction Function

Under parallel update, a conflict occurs when more than 2 agents attempt to move to a same cell. In order to quantitatively describe the level of the conflict, the friction parameter $\mu \in [0, 1]$, which means the probability that all the movements are denied due to friction and clogging effect (Fig. 2), is proposed. Note that through adopting $\mu = 0$ we recover the studies in Ref. [3].

The friction function originates from the assumption that the more agents competing for the chance to move, the stronger the conflict. For example, the impact of friction is stronger when 3 pedestrians are involved in the conflict compared to 2 pedestrians. Yanagisawa et al. conducted simulation to show that most conflicts at the exit cell are among 3 pedestrians [5]. Thus, it is not appropriate to consider only 1 kind of conflict, as the friction parameter does.

The friction function, which takes the number of pedestrians k into consideration, is expressed as follows:

$$\phi_\zeta(k) = 1 - (1-\zeta)^k - k\zeta(1-\zeta)^{k-1} \tag{4}$$

where $\phi_\zeta(k)$ represents the probability the movements of all the involved pedestrians are denied. ζ

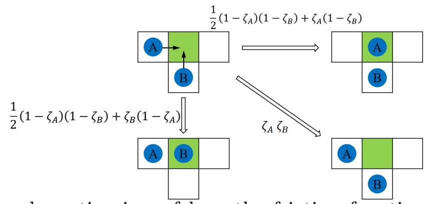
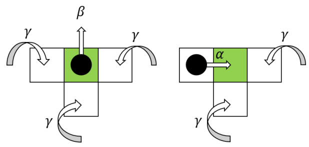
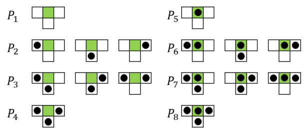


Fig.3: A schematic view of how the friction function resolves conflict. Depending on the strategies of all the involved agents in a conflict, whether the movement is denied is determined.



(a)



(b)

Fig.4: (a) Illustration of notations. α and β represent the transition probability to the exit cell and the transition probability getting out of the room, respectively. γ is the probability an empty neighbor cell be occupied in the next time step. (b) A schematic view of the configuration of exit cell and its Von Neumann neighbor cells. There are $2^4 = 16$ kinds of configurations, which are divided into 8 classes. P_i represents the probability that at a timestep the configuration belongs to class i. Notice that the only difference between the left 4 classes and the right 4 classes is whether the exit cell is occupied.

is an aggressive parameter, which is the probability that a pedestrian does not give ways to others. As shown in Fig. 3, it is supposed that if 2 or more than 2 involved agents do not give ways to others, all the movements will be denied. In other words, only in 2 situations a standstill will not happen: (1) All the agents give ways to others, in this situation an agent will be randomly selected and able to proceed. (2) Only 1 agent does not give ways to others, thus this agent will win the conflict. These 2 situations are expressed by 2 terms: $(1-\zeta)^k$ and $k\zeta(1-\zeta)^{k-1}$, respectively.

3 Simulation and theoretical analysis

3.1 Cluster approximation

In this section, we propose a cluster approximation method to computes the flux of evacuees over exit cell. Since most of the conflicts occur near the exit, we only consider the exit cell and its 3 Von Neumann neighbor cells. The configurations of the 4 cells are shown in Fig. 4.

First we adopt the friction parameter to solve conflicts. We define $P_i(t)$ the probability finding class i at time t, denote the permutation $\dfrac{k!}{(n-k)!}$ as $\binom{n}{k}$, and then take the calculation of P_2 as an example:

$$
\begin{aligned}
P_2(t+1) = & P_1(t)\tbinom{3}{1}\gamma(1-\gamma)^2 + \\
& P_2(t)(1-\alpha)(1-\gamma)^2 + \\
& P_5(t)\beta\tbinom{3}{1}\gamma(1-\gamma)^2 + \\
& P_6(t)\beta(1-\gamma)^2
\end{aligned} \tag{5}
$$

As shown in Fig. 5, the transition into class 2 at time $t+1$ is possible for 4 classes at time t: class 1, class 2, class 5 and class 6. The transition probability of these 4 classes are: $\binom{3}{1}\gamma(1-\gamma)^2$, $(1-\alpha)(1-\gamma)^2$, $\beta\binom{3}{1}\gamma(1-\gamma)^2$ and $\beta(1-\gamma)^2$, respectively.

Similarly, the probability of other classes could be calculated. Furthermore, the master equation of the cluster is listed as follows:

$$
\begin{bmatrix} P_1(t+1) \\ P_2(t+1) \\ P_3(t+1) \\ P_4(t+1) \\ P_5(t+1) \\ P_6(t+1) \\ P_7(t+1) \\ P_8(t+1) \end{bmatrix} =
\begin{bmatrix}
(1-\gamma)^3 & 0 & 0 & 0 & \beta(1-\gamma)^3 & 0 & 0 & 0 \\
3\gamma(1-\gamma)^2 & (1-\alpha)(1-\gamma)^2 & 0 & 0 & 3\beta\gamma(1-\gamma)^2 & \beta(1-\gamma)^2 & 0 & 0 \\
3\gamma^2(1-\gamma) & 2(1-\alpha)\gamma(1-\gamma) & a_{33} & 0 & 3\beta\gamma^2(1-\gamma) & 2\beta\gamma(1-\gamma) & \beta(1-\gamma) & 0 \\
\gamma^3 & (1-\alpha)\gamma^2 & a_{43} & a_{44} & \beta\gamma^3 & \beta\gamma^2 & \beta\gamma & \beta \\
0 & \alpha(1-\gamma)^2 & 0 & 0 & (1-\beta)(1-\gamma)^3 & 0 & 0 & 0 \\
0 & 2\alpha\gamma(1-\gamma) & a_{63} & 0 & 3(1-\beta)\gamma(1-\gamma)^2 & (1-\beta)(1-\gamma)^2 & 0 & 0 \\
0 & \alpha\gamma^2 & a_{73} & a_{74} & (1-\beta)^3\gamma^2(1-\gamma) & 2(1-\beta)\gamma(1-\gamma) & (1-\beta)(1-\gamma) & 0 \\
0 & 0 & 0 & 0 & (1-\beta)\gamma^3 & (1-\beta)\gamma^2 & (1-\beta)\gamma & 1-\beta
\end{bmatrix} \times
\begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \\ P_5(t) \\ P_6(t) \\ P_7(t) \\ P_8(t) \end{bmatrix} \tag{6}
$$

where $a_{33} = (1-\gamma)(\mu_2\alpha^2+(1-\alpha)^2)$, $a_{43} = \gamma(\mu_2\alpha^2+(1-\alpha)^2)$, $a_{44} = \mu_3\alpha^3+3\alpha^2(1-\alpha)\mu_2+(1-\alpha)^3$, $a_{63} = (1-\gamma)(2\alpha(1-\alpha)+\alpha^2(1-\mu_2)$, $a_{73} = \gamma(\alpha^2(1-\mu_2)+2\alpha(1-\alpha))$, $a_{74} = (1-\mu_3)\alpha^3+3\alpha^2(1-\alpha)(1-\mu_2)+3\alpha(1-\alpha)^2$.

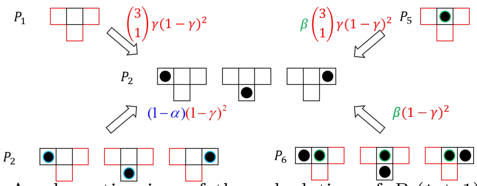


Fig.5: A schematic view of the calculation of $P_2(t+1)$, the color of the relevant part in the graph and the formula are corresponding.

Adding the normalization condition

$$\sum_{i=1}^{8} P_i(t) = 1, \tag{7}$$

the flux (represented by Q) in the stationary state ($t \to \infty$) could be calculated by following equation. For configurations 5, 6, 7 and 8, the agent standing in the exit cell has a probability of β egressing from the room in the next time step.

$$Q = \beta[P_5(t) + P_6(t) + P_7(t) + P_8(t)] \tag{8}$$

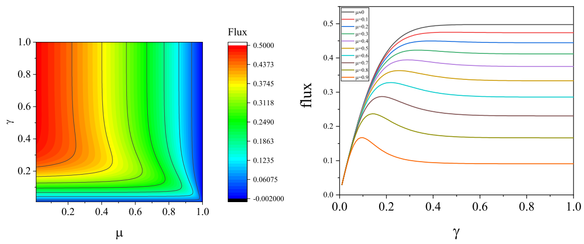
For simplicity we set $\alpha = 1, \beta = 1$. First we validate our model by 3 simple cases, each represents a very special situation where the flux could be easily calculated:

(1) $\gamma = 0$. Because that there is no inflow, whatever the value of μ, we get $P_1 = 1, P_i = 0 (i = 2, 3, ..., 8)$. Apparently the result is correct.

(2) $\mu = 1$. This situation leads to the result: $P_4 = 1, P_i = 0 (i = 1, 2, 3, 5, 6, 7, 8)$, which refers to a complete deadlock condition.

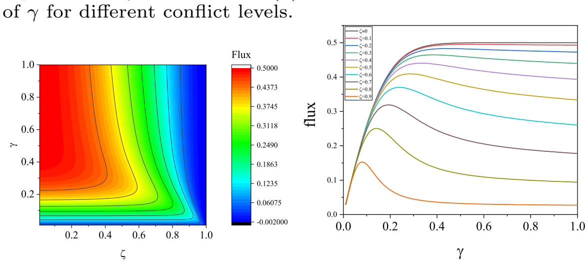
(3) $\gamma = 1$. We get $Q = \dfrac{1-\mu}{2-\mu}$, which is in accordance with the results in previous studies [4] [5] [6].

The calculation result is shown in Fig. 6, from which one can see that as the friction effect being stronger (represented by a larger value of friction parameter μ), the optimal value of γ is declining. We also take μ as fixed variable, and show flux as function of γ. The result is in accordance with



(a) (b)

Fig.6: Calculation results based on the friction parameter. (a) Flux from the exit as function of μ and γ, the high flux regime is represented by warm colors. (b) Flux from the exit as function of γ for different conflict levels.



(a) (b)

Fig.7: Calculation results based on the friction function. The difference from Fig. 6 is that the parameter describing the conflict level is changed to ζ rather than μ. (a) Flux from the exit as function of μ and ζ. (b) Flux from the exit as function of γ for different conflict levels.

the previous work [6], in which the pedestrian flow through multiple bottlenecks is simulated. It has been demonstrated that when the cluster merges faster around the final exit, the overall flow rate declines thus leads to longer evacuation time. According to our result, the final outflow decreases monotonically after the critical point of inflow rate (Fig. 6).

We also present the calculation result based on the friction function, which is easily realized by replacing μ_2 in equation (6) by $1-(1-\zeta)^2-2\zeta(1-\zeta)$ and μ_3 by $1-(1-\zeta)^3-3\zeta(1-\zeta)^2$. The results are shown in Fig. 7.

3.2 Comparison between simulation and theoretical analysis

In the following we present the simulation results to validate the cluster approximation method. In our simulations a representative set of parameters in the so called *ordered regime*: $k_s = 10, k_d = 0, \alpha = 0, \sigma = 0$ is adopted (according to Ref. [3]). We consider a grid of size 63×63 sites with an exit of one cell in the middle of the upper boundary. All the agents are randomly distributed in the room initially.

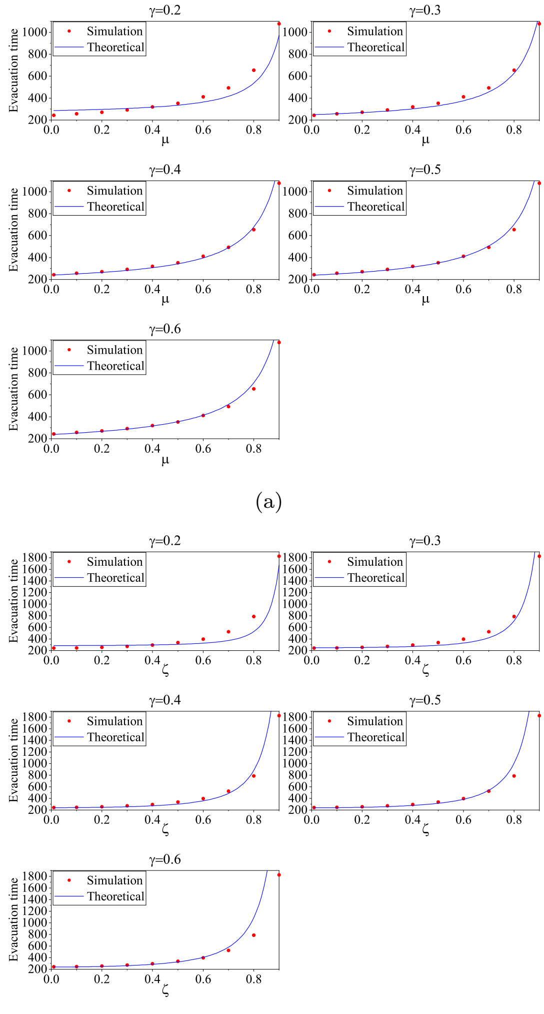
We want to stress that it is not in this research at the first time the flux over exit based on the floor field model is theoretically analyzed. The originality of our work is that the parameter γ is not just a fixed value, to the best of our knowledge, its value is fixed in all the previous works [4] [5] [6]. It is reported in Ref. [4] and Ref. [6] that such setup gives satisfactory agreement for weak friction regime: $\mu \leq 0.6$, but in the region of strong friction: $\mu \geq 0.6$ large deviation can be observed. From Fig. 6 one can clearly observe that the flux takes local minimum value when $\gamma = 1$, that is to say, the reason of the deviation is the underestimate of the flux when $\gamma = 1$ is adopted. What is more, the decline of flux from the optimal point is more obvious as μ grows, thus the deviation in strong friction regime is further enlarged.

The comparison of theoretical analysis and simulation is presented in Fig. 8. Five different values of $\gamma : 0.2, 0.3, 0.4, 0.5, 0.6$ are adopted to calculate the evacuation time. We do not consider the regime: $\gamma \geq 0.6$ because in which the curve slopes in both Fig. 6 and Fig. 7 are not so steep, indicating there may not be huge differences in the calculation results. One can see from Fig. 8 that as γ grows, there is a tendency the theoretical results better fit the weak friction regime: $\mu \leq 0.6(\zeta \leq 0.6)$, but the deviation in the strong friction regime: $\mu \geq 0.6(\zeta \geq 0.6)$ becomes larger. We assume that a moderate value of γ such as 0.3 or 0.4 would minimize the total deviation.

4 Conclusion

In this paper, we have introduced a new method to approximate the outflow from a single cell based on the floor field model. Two different approaches to solve the conflicts are considered and the theoretical results are presented accordingly. We found that despite the difference of conflict level, an optimal inflow rate, which would maximize the overall flux, always exists. The value of the optimal inflow rate tend to decrease as the conflict grows stronger.

We conducted simulations and applied the cluster approximation approach to calculate the evacuation time. We found that for both friction parameter and friction function based simulation, in order to give satisfactory accuracy in strong friction regime, a relatively lower value of the inflow rate should be adopted. The investigation of the inflow rate in stationary state during evacuation will be our future work.



(a)

(b)

Fig.8: The comparison of theoretical analysis and simulation. (a) Results based on the friction parameter. (b) Results based on the friction function.

5 Acknowledgement

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