サイトごとに移動確率の異なる完全非対称単純排他過程における移動確率のばらつきの影響

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概要

サイトごとに二種類の移動確率 p_1 と p_2 のいずれかが割り当てられている開放境界でパラレルアップデートの完全非対称単純排他過程に関して、移動確率のばらつきが流量に与える影響について解析した。二種類の移動確率の配置の仕方として、(1) p_1 と p_2 をそれぞれかたまりで配置、(2) 各々のサイトに p_1 と p_2 を二項分布に従うように配置の 2 通りの場合を考えた。ホップ確率の値のばらつきが流量に与える影響は、境界条件に関わらず、ホップ確率の配置方法によって大きく異なることが明らかになった。

Influence of variation of hopping probabilities on Totally Asymmetric Simple Exclusion Process with site disorder

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Abstract

We investigate the influence of variation of hopping probabilities on current of inhomogeneous Totally Asymmetric Simple Exclusion Process with two hopping probabilities in parallel update under open boundary condition. We study two types of systems with different distribution of disorder : (1) sites are zoned by two different hopping probability subsystems and (2) sites are disordered by binary distribution. The results show that the effect of variation of the value of hopping probabilities differs greatly with allocation method regardless of boundary condition.

1 Introduction

Totally asymmetric simple exclusion process (TASEP) has been widely studied to model non-equilibrium systems such as production flow, vehicular traffic, and biological transport. Since it was revealed that even a single defect on TASEP could lead to global effects on the system [1], theoretical and numerical investigations have been conducted intensively. Especially in traffic systems, jamming phenomena can be attributed to local defects, e.g., narrowing corridors, lane reductions or local speed limits. However, the precise effect of a local defect has not been clarified and it was not until recently that it was revealed whether a finite defect of strength is required to create global effects [2]. While many studies consider a site with lower hopping probability p_2 (< 1) as "defect" compared to the normal sites whose hopping probability p_1 are set to 1, there are few studies that investigate the case of $p_1 < 1$ and the relationship between p_1 and p_2. In this research, we focus on the variation of p_1 and p_2 and study the influence on the current of the whole system. Our interest centers on whether variation effect the current and if it does, how much influence is given. We investigate two types of inhomogeneous TASEP with different allocation method and discuss the differences.

2 Zoned disorder

2.1 Description of the model

Let us first define the model of the disordered TASEP in parallel update under open boundary condition. The original TASEP with input probability α, output probability β and uniform hopping probability p is defined in a 1D lattice of L sites. Each site can hold at most one particle. However, in our zoned disorder model, the system is divided into two segments (Fig.1). The hopping probability of each site in Segment 1 and Segment 2 is p_1 and p_2, respectively. Here, we only investigate the case of $p_1 > p_2$. Throughout this paper we are mainly interested in the current of stationary state in the limit of $L \to \infty$. If the mean $\overline{p} = \frac{p_1+p_2}{2}$ and standard deviation $s = \frac{p_1-p_2}{2}$ of p_1 and p_2 are given, we can get $p_1 = \overline{p} + s$ and $p_2 = \overline{p} - s$.

In order to quantitatively evaluate the effect of the variation of hopping probabilities on the current, we compare the current of system (A), J_{A}, and (B), J_{B} . System (A) is an inhomogeneous TASEP with two hopping probabilities, while system (B) is a homogeneous TASEP with hopping probability $\overline{p} - \Delta p$. By solving $J_{\mathrm{A}} = J_{\mathrm{B}}$, we obtain the relationship between s and Δp, $\Delta p = f(s)$. Thanks to the this formula, we can convert the variation of hopping probability of an inhomogeneous TASEP into a reduction of the hopping probability of a homogeneous TASEP. For example, if $p_1 = 0.7$ and $p_2 = 0.3$ in System (A) and $\overline{p} - \Delta p = 0.4$ in System (B), it indicates that standard deviation of $s = 0.2$ of an inhomogeneous TASEP corresponds to a decrease of hopping probability, $\Delta p = 0.1$, of a homogeneous TASEP with $\overline{p} = 0.5$. In this subsection, we calculate the current and obtain the phase diagram of the above-mentioned inhomogeneous TASEP model. In a homogeneous TASEP with hopping probability p, there are three phases (low-density (LD), high-density (HD), and maximal current (MC)) and current for each phase can be solved exactly as follows [3]: in LD phase $(\alpha < \beta, \alpha < 1 - \sqrt{1-p}), J = \frac{\alpha(p-\alpha)}{p-\alpha^2}$, in HD phase $(\alpha > \beta, \beta < 1 - \sqrt{1-p}), J = \frac{\alpha(p-\alpha)}{p-\alpha^2}$, and in MC phase $(\alpha > 1 - \sqrt{1-p}, \beta > 1 - \sqrt{1-p}), J = \frac{1-\sqrt{1-p}}{2}$. Since Segment 1 and Segment 2 can be considered as a homogeneous TASEP with hopping probability p_1 and a homogeneous TASEP with hopping probability p_2, we can apply the above exact solution of current to each segment. We denote the current of Segment 1 and Segment 2 as J_1 and J_2, respectively. Furthermore, in a stationary state, the current of Segment 1 should be equal to that of Segment 2, $J_1 = J_2$ [4]. For example, when Segment 1 is in LD phase and Segment 2 is in HD phase, $J_1 = \frac{\alpha(p_1-\alpha)}{p_1-\alpha^2}$ and $J_2 = \frac{\beta(p_2-\beta)}{p_2-\beta^2}$. By solving $J_1 = J_2 \quad (= J_{\mathrm{A}})$, we can get conditions of α and β. The current J_{A} (Appendix) of other phases can also be calculated approximately and the phase diagram can be obtained as shown in Fig. 3.

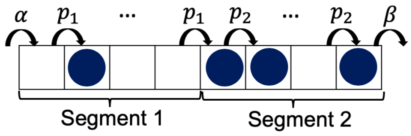

Fig.1: Illustration of an inhomogeneous TASEP with two hopping probabilities. The hopping probability of each site in Segment 1 and Segment 2 is p_1 and p_2, respectively.

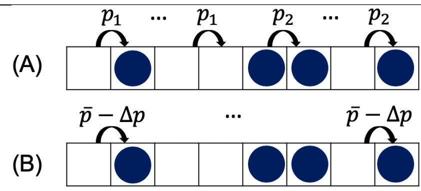

Fig.2: System (A) is a inhomogeneous TASEP with two hopping probabilities p_1 and p_2. System (B) is a homogeneous TASEP with hopping probability $\overline{p} - \Delta p$.

2.2 Results

By using the results of Subsection. 2. 2 and solving $J_{\mathrm{A}} = J_{\mathrm{B}}$, $\Delta p = f(s)$ can be formulated depending on α and β as follows (Fig. 3):

(Area 1) $\Delta p = \overline{p} - \alpha^2 \frac{(\overline{p}-s-\beta^2)-\beta(\overline{p}-s-\beta)}{\alpha(\overline{p}-s-\beta^2)-\beta(\overline{p}-s-\beta)}$

(Area 2) $\Delta p = s$

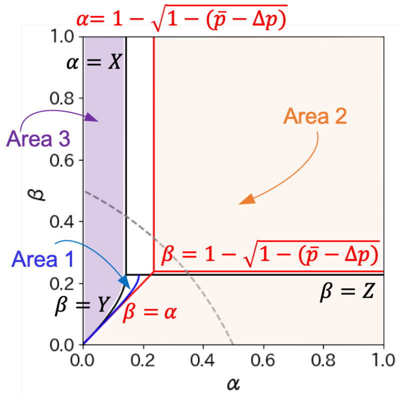

Fig.3: The black lines represent the Phase transition of inhomogeneous TASEP with two hopping probabilities $p_1 = 0.6$ and $p_2 = 0.4$. The red lines represent the Phase transition of homogeneous TASEP with hopping probability $p = 0.418$.

(Area 3) $\Delta p = -s$

In Area 1, the relationship between s and Δp is hyperbolic function. The larger the variation of hopping probability is, the smaller the current of the system is. Therefore, in this area, the variation of hopping probability has direct influence on the current. In Area 2, which is included in the HD phase and the MC phase, Segment 2 with the smaller hopping probability p_2 is the bottleneck and it has a dominant influence on the current. The current of the whole system corresponds to that of a homogeneous TASEP with hopping probability p_2. Contrarily, in Area 3, which is included in the LD phase, even though Segment 2 has smaller hopping probability, it is not the bottleneck of the system and the current corresponds to that of a homogeneous TASEP with hopping probability p_1, the higher hopping probability. This is due to the fact that the higher hopping probability area is near the left edge which particles come into the system. Since the input probability is low in Area 3, the hopping probability on the left edge has crucial influence on the current of the whole system.

In summary, the variation of hopping probabilities has effect on the current directly under very limited conditions, Area 1. In most of the conditions of α and β, Area 2 and Area 3, the current does not depend on the variation of the hopping probabilities but corresponds to the current of a homogeneous system with either p_1 or p_2.

3 Binary distribution disorder

3.1 Description of the model

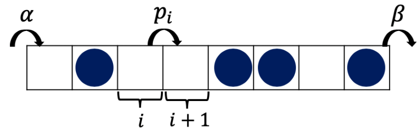

Fig.4: Illustration of an inhomogeneous TASEP with two hopping probabilities distributed to sites by binary distribution function.

The conditions of the model are equivalent to the model described in Subsection. 2. 1 except that two hopping probabilities p_1 and p_2 ($p_1 > p_2$) are allocated to sites by binary distribution function. The hopping takes place with a site-dependent hopping probability p_i which is drawn from a distribution function $f(p)$ (Fig. 4). There is no spatial correlation between the p_i ($i = 1, \cdots, L-1$) and it can be regarded as independent stochastic variables. We denote the binary probability for p_1 and p_2 as $1-f$ and f, respectively. The form of the binary distribution is $f(p) = (1-f)\delta(p-p_1)+f\delta(p-p_2)$ as in [5]. The mean and variance are $\overline{p} = (1-f)p_1 + fp_2$, $s^2 = (1-f){p_1}^2 + f{p_2}^2 - ((1-f)p_1 + fp_2)^2$, respectively. We set the $f = 0.5$ so that the appearance ratio of p_1 and p_2 in the system is 1 : 1 and that the mean, $\overline{p} = \frac{p_1+p_2}{2}$, and variance, $s = \frac{p_1-p_2}{2}$, are same as those of TASEP with zoned disorder. Therefore, the only difference between TASEP with zoned disorder and TASEP with binary distribution is how p_1 and p_2 are allocated to the sites.

3.2 Numerical Analysis

In this subsection, we show the results of the numerical simulations with $L = 300$ sites and averaging has been executed over 100 disordered samples. Fig. 5 is a graph that shows J vs s with LD ($\alpha = 0.05, \beta = 0.8$), HD ($\alpha = 0.8, \beta = 0.05$), MC ($\alpha = 0.9, \beta = 0.9$) when the mean of hopping probabilities are fixed at $\overline{p} = 0.5$ with various standard deviation. Contrary to the case of zoned disorder, it depicts that, in all phases, the effect of increasing the standard deviation is to decrease the current. The decrease of the current of the MC phase exhibits a sharper decrease compared to that of the

LD phase and the HD phase. It indicates that, especially in MC phase, it is better to decrease the variation of the hopping probabilities in order to increase the current. The feature of the LD phase and the HD phase are similar and the s does not have influence on the current while the s is larger than approximately 0.3.

Therefore, we investigate the MC phase intensively. Fig. 6 shows the the graph of J vs p_2 with A (Inhomogeneous system with $f = 0.5$), B (Homogeneous system with hopping probability, $p = 0.5 + 0.5p_2$, and C (Homogeneous system with hopping probability p_2) when $\alpha = \beta = 0.9$, MC phase. We compare the current of an inhomogeneous system, A, and a homogeneous system, B, which has the hopping probability which is equivalent to the mean of the inhomogeous system A, as we examined in subsection 2. 1. It shows that when p_2 is small, the difference of the current of A and B is notably large. As mentioned in Subsection 2. 3, the current of an inhomogeneous TASEP is equivalent to that of a homogeneous TASEP with low hopping probability p_2. Hence, we also draw a comparison between A and homogeneous system, C, with low hopping probability p_2. Although the feature of the increase of current of A and C when $p_2 \to 1.0$ are approximately the same, the current of them are not consistent, contrary to the result of Subsection 2. 3. It indicates that the binary distribution disorder enhances the current compared to zoned disorder.

4 Conclusions

In this study, we investigated inhomogeneous TASEP with two hopping probabilities allocated to sites by two different arranging methods, p_1 and p_2 allocated to sequence of consecutive sites and allocated to each site randomly by binary distribution. We have identified distinctive features between them that the variation of hopping probabilities has large effect on the current when p_1 and p_2 are allocated to sites randomly, but not on that with a zoned disorder.

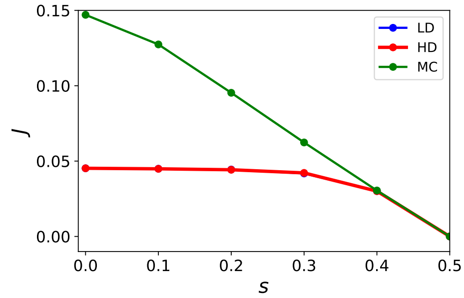

Fig.5: J vs s.

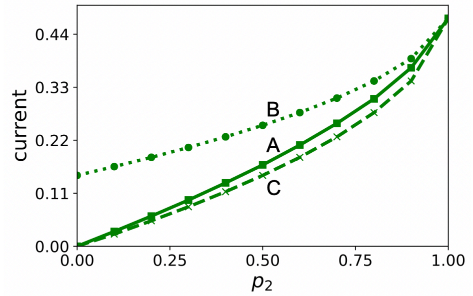

Fig.6: J vs p_2.

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6 Appendix

$J_{\mathrm{A}} = \frac{\alpha(p_1-\alpha)}{p_1-\alpha^2}$ when $\alpha < X$ and $\beta \geq Y$,

$J_{\mathrm{A}} = \frac{\beta(p_2-\beta)}{p_2-\beta^2}$ when $\beta \geq Y$ and $\beta < Z$ and

$J_{\mathrm{A}} = \frac{1-\sqrt{1-p_2}}{2}$ when $\alpha \leq X$ and $\beta > Z$.

$$X = \frac{1}{p_2}\left(p_1 + \sqrt{-p_1\left(p_1p_2 - p_1 - p_2^2 + p_2\right)}\right.$$
$$\left. - \sqrt{-p_1p_2^2 + \left(p_1 + \sqrt{p_1\left(-p_1p_2 + p_1 + p_2^2 - p_2\right)}\right)^2}\right),$$

$$Y = \frac{1}{2p_1(\alpha-1)}\left(p_2(\alpha^2 - p_1)\right.$$
$$\left. + \sqrt{p_2(\alpha^4p_2 - 4\alpha^3p_1 + 4\alpha^2p_1^2 - 2\alpha^2p_1p_2 + 4\alpha^2p_1 - 4\alpha p_1^2 + p_1^2p_2)}\right),$$

$$Z = 1 - \sqrt{1-p_2}.$$

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