高速道路傾斜部周辺での渋滞固着現象の再現と分析

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概要

高速道路の渋滞現象において、渋滞先頭位置は平坦部では後退するが、傾斜部では傾斜部始点周辺で固着することが知られている。しかし、既存のシミュレーションモデルでは、この固着現象を再現することができていない。そこで、ドライバーの行動をもとに、勾配による減速への感知、低速時での安全速度関数と Bang-bang 制御を考慮した追従モデルを提案する。本モデルによって渋滞先頭位置の固着現象の再現に成功し、固着条件を明らかにした。

Reproduction and analysis of traffic-jam fixation around the gradient section

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Abstract

It is well known that although the traffic jam on a flat ground moves backward, the head position of a traffic jam caused by the gradient sections is nearly fixed around the starting point of the gradient section. However, several proposed simulation models have not reproduced traffic-jam fixation. Therefore, we developed a new traffic model that reproduces traffic-jam fixation. We employed the optimal velocity model and introduced the bang-bang acceleration in the low-velocity condition (driving in traffic jam). The proposed model reproduces the fixation of the head position of the traffic jam around the starting point of the gradient sections. We also analysed the traffic-jam fixation conditions.

1 Introduction

Many traffic jams occur in gradient sections in Japan. The reason of the traffic jam in gradient sections is that the traffic flow is lower than in flat sections. It is well known that the traffic jam on a flat ground moves backward and form stop-and-go waves when traffic density is sufficiently high. On the other hand, the head position of a traffic jam caused by the gradient section is nearly fixed around the starting point of the gradient section [1]. In other words, the boundary position of congestion almost stops at the same position for a long time.

Several simulation models have been proposed for the reproduction of the traffic jam in the gradient section [2, 3]. However, these model do not focus on traffic-jam fixation in the gradient section.

In this research, we present a new traffic model that aims to reproduce the traffic-jam fixation in the gradient section. We propose new realistic concepts of the simulation model, and confirm the traffic-jam fixation.

2 Optimal velocity model

Optimal velocity (OV) model is one of the mathematical models which is often used in modeling traffic flow [4]. The position of the nth vehicle $x_n(t)$ at the time t is calculated by the following equation:

$$\ddot{x}_n(t) = a_{\text{OV}}(V_{\text{opt}}(\Delta x_n) - \dot{x}_n), \ \Delta x_n = x_{n+1} - x_n, \tag{1}$$

where Δx_n is the headway distance of the *n*th vehicle, $a_{\rm OV}$ is the coefficient of sensitivity, and $V_{\rm opt}$ is the optimal velocity function dependent on the headway distance.

In this conventional OV model, the effect of bottlenecks (gradient sections) and the behavior of vehicles traveling at low velocity in the traffic jam are not introduced. Therefore, we need to propose a new model in order to reproduce detailed behavior of vehicles in gradient sections.

3 Gradient optimal velocity model

We take into consideration the influence of the gravitational acceleration and drivers' realization effect of the deceleration in the gradient sections. Furthermore, we introduce the bang-bang acceleration and the safety velocity function to model the acceleration in the low velocity condition.

3.1 Gravity effect and drivers' realizing effect of the gentle gradient

The vehicle moving on the gradient section without realizing it decelerates by the gradient effect. We consider two states of vehicles based on the drivers' realization effect, which are before realizing state and after realizing state. The vehicles in the before realizing state decelerate by the gradient effect when they are moving on the gradient section. However, they do not aware of the deceleration. Therefore, the actual position of the vehicle x and the drivers' expecting position of the vehicle \hat{x} become inconsistent. The motion of the equations for x and \hat{x} are described as follows:

$$\begin{cases} \ddot{x}_n(t) = F(\Delta x_n, \hat{x}_n) - g\sin\theta \\ \hat{x}_n(t) = F(\Delta x_n, \hat{x}_n) \end{cases}$$
(2)

where F is the acceleration rule, θ is the slope of the gradient, and g is the gravitational acceleration.

We propose the effect that drivers realize the deceleration in the gentle gradient. In before realizing state, the vehicle decelerates by the gravitational acceleration, and the difference between the real velocity and the expected velocity gets larger. The vehicle can realize the deceleration by the gradient when the difference between the real velocity and the expected velocity exceeds the realizing velocity difference V_r . That is, the condition to become after realizing state is formulated as follows:

$$\hat{\dot{x}}_n - \dot{x}_n = \int g \sin \theta(x_n) dt > V_r. \tag{3}$$

When vehicles become after realizing state, the expected velocity \hat{x}_n gets equal to the real velocity \dot{x}_n . The vehicle in after realizing state is not affected by the gradient effect, that is, the velocity is calculated in the same formula as the flat section ($\theta = 0.0$). We set $V_r = 10 \text{(km/h)}$ based on the experiment data.

3.2 Bang-bang acceleration and safety velocity function

Vehicles can not accelerate continuously on initial movement from the viewpoint of the performance of the engine, and the drivers avoid the continuously changing acceleration which causes car sickness [5]. For these points, the acceleration of a vehicle in the low-velocity is not changed continuously. Thus, we introduced the bang-bang acceleration [6] in the low-velocity condition as follows:

$$F(\Delta x_n, \hat{x}_n) = a_{\text{bang}} \operatorname{sgn}(V_{\text{opt}}(\Delta x_n) - \hat{x}_n), \tag{4}$$

where a_{bang} is the acceleration in the low-velocity condition based on the experiment data.

In traffic jam, drivers need to consider the situation that forward vehicle stops at its place suddenly to avoid traffic accidents. Thus, we introduce safety velocity function, which represent the limit of the stoppable velocity even if forward vehicle stops at the current position suddenly. When we assume that the acceleration of the sudden braking is constant, the motion equation of a vehicle in the sudden deceleration condition is formulated as follows:

$$\begin{cases} \dot{x}_n(t) - \int_t^{t+t_{\text{safe}}} a_{\text{safe}} d\tau = 0\\ \Delta x_n(t) - l \ge \int_t^{t+t_{\text{safe}}} (\dot{x}_n(t) - a_{\text{safe}} \tau) d\tau \end{cases}$$
(5)

where a_{safe} represents the constant acceleration of the sudden braking, t_{safe} is the necessary time to decelerate, and l is the length of vehicles. The safety velocity function is defined as the limit of the stoppable velocity. This critical velocity can be calculated from equation (5) as follows:

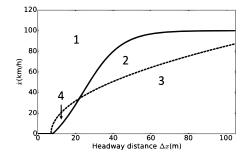
$$\dot{x}_n \le \sqrt{2a_{\text{safe}}(\Delta x_n - l)} = V_{\text{safe}}(\Delta x_n). \tag{6}$$

3.3 Acceleration rule

By using V_{opt} and V_{safe} , we divide the headway-velocity plane into four categories as in Figure(1). We propose the acceleration rule in each category by considering the OV acceleration and the bang-bang acceleration as follows:

$$F(\Delta x_n, \hat{x}_n) = \begin{cases} a_{\text{OV}}(V_{\text{opt}}(\Delta x_n) - \hat{x}_n) & (\hat{x}_n \ge V_{\text{opt}}, \hat{x}_n \ge V_{\text{safe}}) \\ a_{\text{OV}}(V_{\text{opt}}(\Delta x_n) - \hat{x}_n)(1 - e^{-k\hat{x}_n}) + a_{\text{bang}} \text{sgn}(V_{\text{safe}}(\Delta x_n) - \hat{x}_n)e^{-k\hat{x}_n} & (\hat{x}_n < V_{\text{opt}}) \\ + a_{\text{bang}} & (\hat{x}_n \ge V_{\text{opt}}, \hat{x}_n < V_{\text{safe}}) \end{cases}$$
(7)

We determined k from $e^{-kv_{\rm jam}} \approx 0.5$, where $v_{\rm jam} = 30$ km/h. When \hat{x}_n is smaller than $V_{\rm opt}$ (Categories 2, 3), vehicles accelerate and decelerate based on the OV acceleration and the Bang-bang acceleration. The ratio of the Bang-bang acceleration increases, when the velocity decreases by the term of $e^{-k\hat{x}_n}$ because drivers have to prepare for the situation that forward vehicle stops at the current position suddenly. In Category 1, vehicles decelerate based on the OV acceleration in order to avoid collision because $\hat{x}_n > V_{\rm opt}$ and $\hat{x}_n > V_{\rm safe}$. In Category 4, vehicles do not accelerate based on the velocity which drivers consider to be optimal but accelerate in Bang-bang acceleration because $V_{\rm safe} > V_{\rm opt}$.



 \boxtimes 1: The relation between the optimal velocity function and the safety velocity function. Solid line and dotted line indicate $V_{\rm opt}$ and $V_{\rm safe}$, respectively. Each number represents the category of the acceleration rule.

4 Simulation results

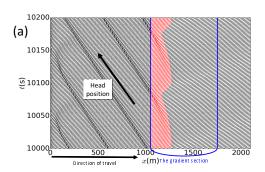
The simulation results in $\tan \theta = 3.0\%$ are shown in Fig.(2). $k \to \infty$ means that Bang-bang acceleration is not considered at all. In Fig.(2,a) $k \to \infty$ (Without Bang-bang acceleration), traffic jam happens, but the head position of the traffic jam is not fixed. In Fig.(2,b) k = 0.05 (With Bang-bang acceleration), the head position of the traffic jam is fixed around the starting point of the gradient section. Thus, Bang-bang acceleration in the low velocity condition can reproduce the traffic-jam fixation.

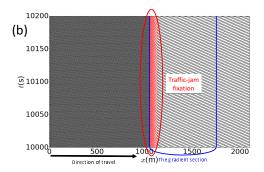
5 Summary

We developed Gradient optimal velocity model and reproduce the traffic-jam fixation as follows:

- 1. The deceleration in the gradient section is reproduced by the introduction of the expected velocity.
- 2. Vehicles do not accelerate enough by the gradient effect and the bang-bang acceleration around the head position of traffic jam.
- 3. The following vehicle proceeds too close to the accelerating vehicle, which is affected by the gradient effect and the bang-bang acceleration, and decelerates based on the acceleration rule.
- 4. The head position of traffic jam stays at the almost same position.

We would also like to mention that all the parameters are based on real data. Furthermore, we confirmed that slight change of the parameters does not affect the traffic-jam fixation.





 \boxtimes 2: Trajectories of vehicles (space-time diagram) for $\tan \theta = 3.0\%$. (a) $k \to \infty$ (Without Bang-bang acceleration) and (b) k = 0.05 (With Bang-bang acceleration). Blue lines represent the starting point and the end point of the gradient section. The red trajectories and the black trajectories represent vehicles which do not realize the gradient effect, and vehicles which are not affected by the gradient effect respectively.

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