トーナメントの問題

徳山喜一, 前村隆太, 横内健悟, 大平徹

名古屋大学 大学院多元数理科学研究科

概要

サッカーやテニスのように、スポーツの世界ではチームや選手の強さにランク付けがなされている場合がある。これらの選手がトーナメントで試合をした場合の個々の結果の順位は、当然それらのランキングのとおりにはならないが、統計的にみて見た時にはある程度は反映されているだろうか。本論文ではこの問題に数理的な方向から取り組む。具体的には強さのランク付けがされた選手によるモデルトーナメントを考える。さらに、現実を反映して、必ずしも高いランクの選手が勝利しない適度な確率を導入する。すると、トップランクの選手だけではなく、上位ランクの何人かの選手においても、これらの選手が実際に 1 位になる確率は、トーナメントのより下位の順位に終わる確率よりも大きくなるという現象がみられた。例えば、ランキングが二番の選手でも、トーナメントで 1 位になる確率のほうが、２位に終わる確率よりも高くなる。この「逆転現象」は、ランクの強いほうが必ず勝つという設定では現れない。また、この現象は現実のテニスのトーナメントでもその存在が示唆されたので報告する。

Ranking and Winning in Tournaments

Kichi Tokuyama, Ryuta Maemura, Kengo Yokouchi, Toru Ohira

Graduate School of Mathematics, Nagoya University, Nagoya, Japan

Abstract

In some sports such as tennis and soccer, there are rankings of teams or players. We pose here a question of how ranking of players and their performance in tournaments are related. In particular, we consider a mathematical model tournaments among players of different rankings. It turns out there is a rather unexpected feature of relation between rankings and winning in tournaments We found that the probability to reach the top position is higher than that of finishing up at lower positions for not only the number one ranked player, but also for a range of top players. For example, the second ranked player can have higher probability of finishing at the first than the second position in tournaments. This "inversion characteristics" are shown to be observed with simple mathematical model tournaments as well as in the real tournaments.

1 Introduction

Tournaments are commonly used in sports and other games. In some sports, such as tennis, there are rankings of players entering into tournaments. It is one of interests of spectators how rankings of players and tournament results compare. Even though there believed to be certain correlations between rankings and winning orders in tournaments, no clear picture has been drawn. By formulating this problem into a mathematical framework, we have found a rather peculiar and counter-intuitive general characteristics: the probability of winning a tournament is highest, compared to that of placed at lower positions, not only for the top ranked player, but also for other high ranked players.For example, the second ranked player can have higher probability of finishing at the first than the second position in tournaments. There is an indication that this observation of "inversion characteristics" is true from the results of real tournaments[1, 2].

2 Model

Let us start by explaining our simple mathematical model tournament with N players. The shape of tournaments is the usual "binary tree-like" with the winner advancing to the next level. We give each of our players a set (r, s) of a "rank" r and "strength" s. At each game in a tournament, the winning probability of a player is set proportional to his relative strength against his opponent. In concrete, in a match by two players A and B with strength of s_A and, s_B respectively, we give the winning probability for player A equal to $p(A) = \frac{s_A}{(s_A+s_B)}$, and similarly for player B. (We assume no draw.)

As a first step, we consider the case in which each player has a rank and a strength of $(r, N-r+1)$, $r = 1, 2, 3, \ldots, N$. Through combinatorial calculations, we investigated how each player in a tournament finishes. The probabilities for a player to become first, second, third, or fourth places in tournaments against his rank are plotted in Figure 1.

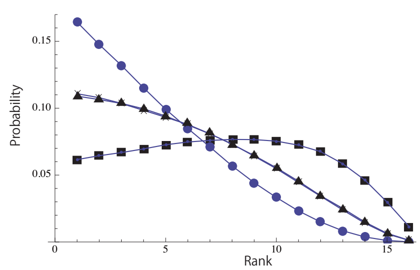

図 1: The result of the model tournament with $N = 16$ players. The probabilities to finish at the first (dot), second (X), third (triangle), or fourth (square) places are plotted for each player against his rank.

The player ranked at the top has the best chance of winning the first place in the tournament, which is as expected. However, the most notable and counter-intuitive point is that for the second to fifth ranked players, their chance of reaching the first place is higher than that for them to be placed at positions according to their ranks. For example, for the third ranked player, the chance that he wins the tournament is better than for him to finish at the second or third places.

Table 1 show that, for different size of tournaments, the range of higher ranked players who have the probability to win the first place higher than that of their becoming of other positions. We see that certain ranges of top players have this "inversion characteristics" in the model.

3 Real Tournaments

We can calculate to show that this observed inversion characteristics is not true if the rule is changed in an unrealistic way so that the higher ranked player always wins in a match. In reality, however, details of winning and losing probabilities in matches vary, and it may be that these inversion characteristics are observed commonly in real tournaments. In order to test this hypothesis, we have investigated on real tennis tournaments. Data sets are obtained through the rankings and match results of Association of Tennis Professionals (ATP) [3] and Women's Tennis Association (WTA) [4] The result is shown in Figure 2. Even though statistics are not enough, we can observe the similar inversion characteristics, indicating our hypothesis has certain validity.

4 Discussions

There are couple points to note. First, we have also considered a hybrid-case where winning and losing probabilities of national football teams with different strength measured in "FIFA points"[2]. Statistics are compiled with data of matches from Federation of International Football Association (FIFA) [5]. Based on this statistics, we performed a hypothetical tournaments by computer simulations. The inversion characteristics are also observed indicating that, regardless of details of winning and losing probabilities in matches, these inversion characteristics are observed commonly in

N	4	8	16	32	64	128	256	512	1024
R	1-2	1-3	1-5	1-9	1-16	1-29	1-55	1-95	1-178

表 1: Ranges R of top players who have inversion characteristics with varying size N of tournaments.

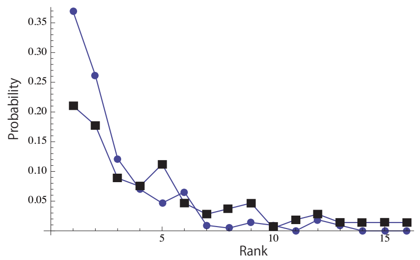

図 2: A result from real tennis tournaments. Data are taken and compiled from 214 tournaments with 16-seedings over the period of 1952 to 2001. The probabilities to finish at the first (dot), or second (square) places are plotted for each player against his/her rank.(The tournaments did not have matches to decide on the third place.)

various tournaments Secondly, in the real tournaments, including the ones shown in Figure 2, we have stronger players placed in certain positions, i.e., seeding. Seedings make the higher ranked players more advantageous in tournaments. Detailed mathematical investigation of such effects is left for further research. We also, note that the probabilities to be second or third places are close together in our simple mathematical model and simulations. This is related to the fact both are the results of losing one game with the same number of matches in a tournament. As many tournaments do not have the third place match, we have not yet investigated whether this can also be seen in the real tournaments.

We have presented simple models of pedestrian counter flows and tournaments. Though simple, they have exhibited rather counter intuitive characteristics. More theoretical investigations are needed to uncover the mechanism of these behaviors.

参考文献

[1] K. Tokuyama, R. Maemura, K. Yokouchi, T. Ohira: arXive:1309.4587 (2013)

[2] K. Tokuyama, R. Maemura, K. Yokouchi, T. Ohira: in *Proceeding of the Annual Meeting of the Japan Society for Industrial and Applied Mathematics*, 9170 (Fukuoka, Japan, 2013)

[3] ATP Official Web Site: http://www.atpworldtour.com

[4] WTA Official Web Site: http://www.wtatennis.com

[5] FIFA Official Web Site: http://www.fifa.com