Swarm oscillators モデルの円状パターン、及びその周辺相

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Abstract

内部自由度と空間自由度を持つ素子群が相互作用によって構造創発する系として、自己駆動型結合振動子系の一つである swarm oscillators モデルを解析する。このモデルでは、振動素子群が、四つの実パラメータやシステムサイズ、素子数に応じて、様々な時空パターンを形成する。本論では、それらパターンの中でも空間二次元における円状パターンに着目する。このパターンでは、ほとんどの素子はダイナミックに変化する円状に配置し、残った少数の素子がその円に囲まれるという複雑な挙動を示す。我々はこのパターンが観察されるパラメータを特定し、その形成メカニズムを説明する。またこのパターンは、あるパラメータ範囲では、安定なリミットサイクル軌道に漸近する。このとき、素子群は同心円上に配置され、対称性の良いパターンを成す。

Circular pattern and its neighboring phases in swarm oscillators model

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Abstract

Swarm oscillators model is a normal form of a general chemotactic model of oscillators. Each oscillator in this model has internal and spatial degrees of freedom, and there exists a nonlinear coupling between oscillators that depends on their internal states. The model exhibits a rich variety of spatio-temporal patterns depending on its four real-parameters, the system size and the number of oscillators. In this paper, we concentrate on a circular pattern. In this pattern, some oscillators exhibit a circular alignment, and the other oscillators are surrounded by the circular alignment. We study a parameter region where the pattern is observed, and we explain the formation mechanism of the pattern. We also report that the pattern becomes a symmetric steady final state in a particular set of parameters.

1 Introduction

A system constructed of many simple elements, such as bacteria [1], self-driven particles [2], and nodes on network, can exhibit several collective patterns. These patterns have collective functions of the system, such as chance for survival of cellular slime molds, ant colony optimization, and swarm intelligence of many simple robots. Collective behavior provide both of robustness and flexibility for the systems. In this paper, we study a model of motile oscillators, which we call swarm oscillators model [3][4]. The model was derived from a general chemotactic model of oscillators by the means of center manifold reduction and phase reduction methods at a multiple bifurcation point:

$$\dot{\psi}_i = \sum_{i\neq j} e^{-|\boldsymbol{R}_{ji}|}\sin(\Psi_{ji}+\alpha|\boldsymbol{R}_{ji}|-c_1), \tag{1}$$

$$\dot{\boldsymbol{r}}_i = c_3\sum_{i\neq j} \hat{\boldsymbol{R}}_{ji} e^{-|\boldsymbol{R}_{ji}|}\sin(\Psi_{ji}+\alpha|\boldsymbol{R}_{ji}|-c_2). \tag{2}$$

ψ_i : Phase of oscillator i
$\boldsymbol{r}_i$: Position vector of oscillator i
$\Psi_{ji} \equiv \psi_j - \psi_i$: Phase difference between oscillator i and j
$|\boldsymbol{R}_{ji}| = |\boldsymbol{r}_j - \boldsymbol{r}_i|$: Distance between oscillator i and j
$\hat{\boldsymbol{R}}_{ji} \equiv \frac{\boldsymbol{R}_{ji}}{|\boldsymbol{R}_{ji}|}$: Unit vector of $\boldsymbol{r}_j - \boldsymbol{r}_i$

The model exhibits a rich variety of spatio-temporal patterns depending on the four real parameters c_1, c_2, c_3, α, the system size L, and the number of oscillators N. In this study, we concentrate on a circular pattern observed in two-dimensional space in the neighborhood of $c_1 = 1.3, c_2 = 3.0, c_3 = 0.02, \alpha = 0$ (fig.1). In this pattern, some outer oscillators exhibit a circular alignment, and the other inner oscillators are surrounded by the circular alignment. Spatial position of all oscillators varies with time. The outer oscillators and inner oscillators partly replace each other. The outer circular alignment exhibits a rather symmetric circle for a while (typically, left top of fig.1), then a rather collapsed circle (typically, right bottom of fig.1), and repeat it with time. We call this pattern 'dynamic circular pattern' or briefly 'DC pattern' in the following.

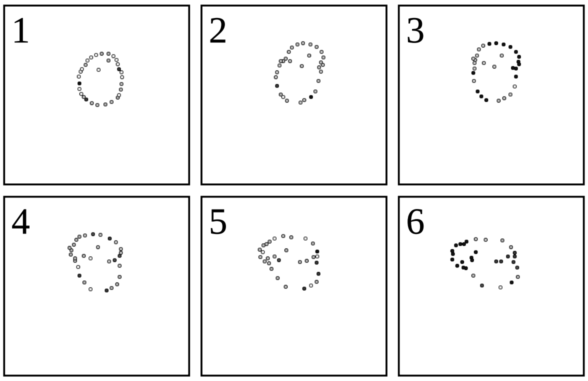

Figure1: Typical snapshots of 'dynamic circular pattern' (DC pattern). Gray scale represents phase of oscillators. 1:$t = 0$, 2: $t = 25$, 3: $t = 50$, 4: $t = 75$, 5: $t = 100$, 6: $t = 125$. $N = 30$.

2 Neighboring phases

In order to study the formation mechanism of DC pattern, we draw two phase diagrams on the parameter space of c_1 and c_2 by using numerical simulations. First and second phase diagrams are drawn by using the order parameter 'variance of phase' $< (< \psi_i > - \psi_i)^2 >$ and 'average of distance to the nearest oscillator' $< \min_j |\boldsymbol{r}_i - \boldsymbol{r}_j| >$ respectively, where the brackets $< \cdots >$ denote average over label of oscillators i and time t. To ignore the effect of transient, we use the average time from $t = 5000$ to $t = 10000$. The other conditions are as follows: number of oscillators $N = 20$ and system size $L = 10$ with periodic boundary conditions. The two phase diagrams are shown in fig.2. By using combination of whether the oscillators synchronize and of whether the oscillators are distributed in the entire space, the parameter space of c_1 and c_2 is roughly divided to four regions labeled by A-D with the boundaries $c_1 = \pi/2, c_1 = 3\pi/2, c_2 = 0$, and $c_2 = \pi$. The labels of A, B, C, and D correspond to typical patterns A, B, C, and D respectively shown in fig.3.

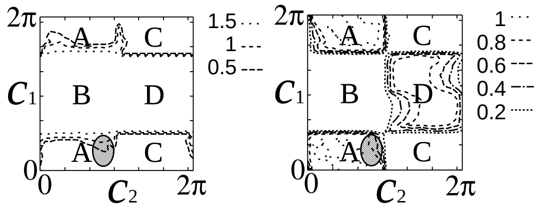

Figure2: Phase diagrams with contour lines of the value of (a) variance of phase and (b) average of distance to the nearest oscillator. DC pattern is observed in the neighborhood of shaded regions. The labels of A, B, C, and D correspond to typical patterns shown in fig.3.

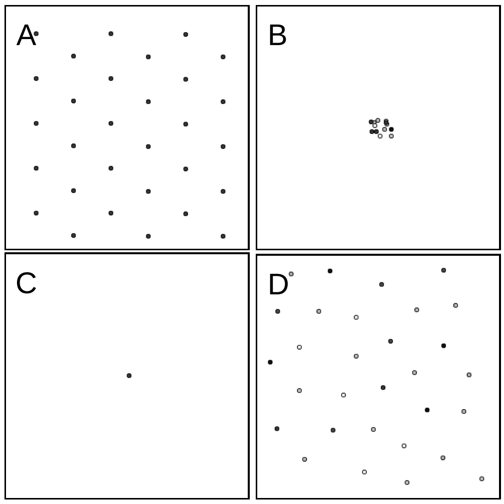

Figure3: Typical patterns in region A($c_1 = 0.5, c_2 = 1.5$), B($c_1 = 3.0, c_2 = 2.0$), C($c_1 = 6.0, c_2 = 4.0$), and D($c_1 = 4.0, c_2 = 4.0$). $c_3 = 0.02, \alpha = 0$. $N = 30$. In the patterns A and C, the oscillators synchronize. In the patterns B and D, the oscillators do not synchronize. In the patterns A and D, the oscillators are distributed in the entire space, but in C and D, they do not.

Although many-body problems are difficult in general, analyzing two-oscillator system, we explain how the parameter space is divided to the four regions. First, we consider the phase difference between the two oscillators $\Psi \equiv \psi_2 - \psi_1$. Substituting $\alpha = 0$ into eqs.(1) and (2), we obtain the equation

$$\dot{\Psi} = -2e^{-R} \sin\Psi \cos c_1. \tag{3}$$

This equation implies that the interaction of phase is attractive (repulsive) when $\cos c_1$ is positive (negative). In fact, the oscillators are synchronized in the regions A and C (desynchronized in the regions B and D) where $\cos c_1$ is approximately positive (negative). Next, we consider the distance between the two oscillators $R \equiv x_2 - x_1$, where $x_i \equiv \hat{\boldsymbol{R}}_{21}|_{t=0} \cdot \boldsymbol{r}_1$. The dynamics of R obeys the equation

$$\dot{R} = 2c_3 e^{-R}(R/|R|) \cos\Psi \sin c_2. \tag{4}$$

This equation implies that the interaction of position is attractive (repulsive) when $\cos\Psi \sin c_2$ is negative (positive). Because the sign of $\cos\Psi$ is equal to that of $\cos c_1$ as shown in eq.(3), the oscillators aggregate (separate) when $\cos c_1 \sin c_2$ is negative (positive), which is approximately the case of the the regions B and C (A and D).

3 Dynamic circular pattern (DC pattern)

We explain how DC pattern is exhibited in the parameter region A by using the above results of two-oscillator system. DC pattern sustains its dynamical behavior repeating the following four steps (1)-(4). In this parameter region, the nearby oscillators tend to synchronize, and a synchronized cluster appears (1). Because the synchronized oscillators tend to separate, some oscillators on an edge of the cluster separate from the cluster (2). The separate oscillators are desynchronized due to the interaction that decays exponentially with the distance (3). The desynchronized oscillators return to the cluster that moves randomly in space (4).

4 Static circular pattern (SC pattern)

In a particular set of parameters, DC pattern becomes a symmetric steady final state (fig.4), which we call 'static circular pattern' or briefly 'SC pattern'. In this pattern, some outer oscillators exhibit a circular alignment, and the other inner oscillators are surrounded by the circular alignment, which is the same as in DC pattern. The difference of SC pattern from DC pattern is such that the spatial distribution of oscillators is static and symmetric. In SC pattern, the spatial distribution of oscillators, the diameter, and the number ratio between the outer and inner oscillators, vary depending on initial values even if the number of oscillators and the parameters are the same. For example, we show two different patterns in fig.4, which are obtained under the same conditions other than initial values. The outer and inner oscillators are

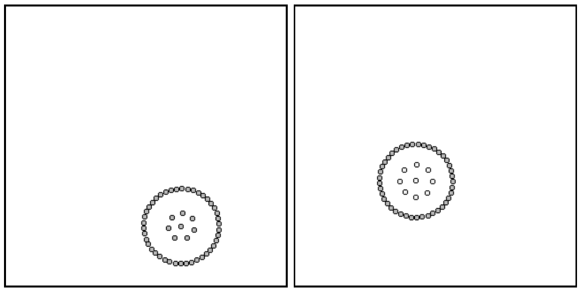

Figure4: Typical 'static circular pattern' (SC pattern). $c_1 = 1.3, c_2 = 3.0, c_3 = 0.02, \alpha = 0, N = 50$. The number ratio between the outer and inner oscillators is different depending on initial values; $42/8$ in the left and $41/9$ in the right.

entrained with a constant phase difference θ. This θ is approximately proportional to the number ratio $k \equiv$(Num. of outer osc./Num. of inner osc.) shown in fig5(a). The interactions between outer oscillators and between inner oscillators are repulsive. In contrast, the interactions between outer and inner oscillators are asymmetric such that the outer (inner) oscillators exert a repulsive (attractive) force to the inner (outer) oscillators as illustrated in fig.5(b). All these interactions balance, and the oscillators are spatially fixed in symmetric distributions.

In order to study the formation mechanism of SC patterns, we concentrate on simpler SC patterns in which the number of inner oscillator is one (fig.6). Because $c_3 = 0.02 \ll O(1)$, $|\dot{\boldsymbol{r}}_i| (= O(c_3))$ is much smaller than $|\dot{\psi}_i| (= O(1))$. (Note that all variables and parameters are dimensionless quantities because the model, eqs.(1) and (2), are normalized [3].) Using this fact, we ignore the time derivative of position of oscillators, i.e., we approximate the dynamics of phase of outer and inner oscillators as $\dot{\psi}_o = -R_{io}\sin(\theta + c_1) - R_{oo}\sin c_1, \dot{\psi}_i = R_{oi}\sin(\theta - c_1)$, where ψ_o and ψ_i represent phase of outer and inner oscillators respectively, $R_{io} \equiv \sum_{\text{inner osc.}\, j} e^{-|\mathbf{R}_{jo}|}$, $R_{oo} \equiv \sum_{\text{outer osc.}\, j} e^{-|\mathbf{R}_{jo}|}$, $R_{oi} \equiv \sum_{\text{outer osc.}\, j} e^{-|\mathbf{R}_{ji}|}$. $\theta = \psi_o - \psi_i$ obeys

$$\dot{\theta} = R_{io}\sin(-\theta - c_1) - R_{oi}\sin(\theta - c_1) + R_{oo}\sin(-c_1). \tag{5}$$

Substituting the numerically observed values $R_{io} = 0.25, R_{oo} = 1.05, R_{oi} = 2.03$ to eq.(5), we find unique stable fixed point $\theta = 0.53$. This value is equal to the numerically observed value $\theta = 0.53$. (Here we observe all numerical values to two places of decimals.) This agreement supports that the above semi-analytical argument sheds light on the formation mechanism of SC pattern: Once DC pattern exhibits an approximately symmetric distribution of oscillators, the phases of oscillators are rapidly entrained satisfying a stable value of θ. This rapidness is due to the fact that the phase velocity is much larger than the spatial velocity ($c_3 \ll O(1)$). In this way, SC pattern is exhibited, in which asymmetric interactions between oscillators balance. However, it remains the unsolved problem of when and how DC pattern exhibits an approximately symmetric distribution of oscillators. Although this should be related sensitively to the parameter values, we do not yet reveal it. For example, for the parameter values $c_1 = 1.3, c_2 = 3.0, c_3 = 0.02, \alpha = 0.0$, SC pattern is exhibited within about $t = 20000$. In contrast, for the slightly changed parameter values $c_1 = 1.33, c_2 = 2.92, c_3 = 0.02, \alpha = 0.0$, SC pattern is not exhibited at least by $t = 10^7$, and DC pattern seems to be final state in this case. Preliminary results suggest that the parameter region for DC pattern is narrower than that for SC pattern (fig.7).

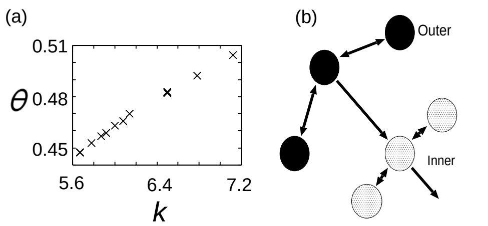

Figure5: (a) Approximately proportional relation between the phase difference θ and the number ratio k. (b) Interactions in SC pattern. Filled and checked circles represent the outer and inner oscillators respectively (see text).

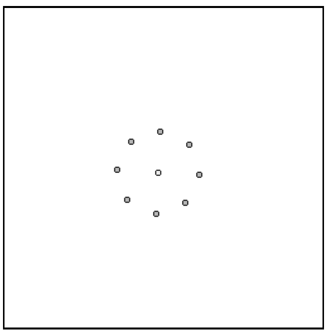

Figure6: A simpler SC pattern. $N = 9$.

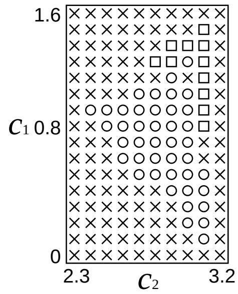

Figure7: Phase diagram. □ for DC pattern , ○ for SC pattern , and × for the others . $c_3 = 0.02, \alpha = 0.0, N = 30$. We use tentatively $t = 20000$ as a transient time.

5 Conclusion

We study 'dynamic circular pattern' (DC pattern) exhibited by a model of motile oscillators, which we call swarm oscillators model [3]. We show phase diagrams on the parameter space of c_1 and c_2, which is roughly divided to four parameter regions. DC pattern is observed in the neighborhood of boundaries of the parameter regions. Analyzing two-oscillator system, we explain how the parameter space is divided to the four regions. DC pattern is exhibited due to a feedback loop in interactions between oscillators. In a particular set of parameters, DC pattern asymptotically approaches to a symmetric steady final state which we call 'static circular pattern' (SC pattern). SC pattern is exhibited due to a balance of asymmetric interactions between oscillators.

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