Understanding "synchronized flow" by optimal velocity model

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Abstract. We perform the simulations of optimal velocity model with bottleneck and reveal the new aspects of this model under such boundary conditions, which provides the interesting possibility of mathematical understanding of "synchronized flow". The "oscillatory wave" solution, which appears in the intermediate distance from the bottleneck, is followed by the jam flow. The wave has the characteristics which can be understood as the "synchronized flow".

The traffic flow offers many interesting problems of physics for collective phenomena in non-equilibrium dynamics [1, 2]. Recently, the phenomenon of traffic flow in a bottleneck, so called "synchronized flow" affects many researchers [3]. It is a challenging problem to provide the mathematical understanding on the view point of non-equilibrium physics. In this paper, we perform the simulations of optimal velocity (OV) model [4] with bottleneck and investigate the behaviors of this model under such boundary conditions for the purpose of providing the possibility of mathematical understanding of "synchronized flow".

OV model is one of the most successful models for describing the dynamics of highway traffic [4]. The model is very simple and formulated as

$$\frac{d^2}{dt^2}x_n(t) = a\left\{V(\Delta x_n) - \frac{d}{dt}x_n(t)\right\}, \tag{1}$$

where x_n is the position of the nth car, and $\Delta x_n = x_{n+1} - x_n$ is the headway. The parameter a is a sensitivity constant. $V(\Delta x_n)$, so called OV-function, determines the optimal velocity according to the headway, which should have the form such as

$$V(\Delta x) = V_0\{\tanh(\Delta x - c) + const.\}. \tag{2}$$

The model has the homogeneous flow solution; cars are uniformly distributed and moving with the same velocity $V(b)$, where b is the average density of vehicles. The homogeneous flow is linearly unstable under the condition $\frac{d}{dx}V(x)|_{x=b} > a/2$. In this case, the jam flow is dynamically formed in the simulation on the circuit. After relaxation, all jam clusters are moving with the same velocity opposed to the direction of the

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vehicle. The profile of jam flow solution is recognized as the trajectory of vehicles in the phase space of headway and velocity $(\Delta x_n, \dot{x}_n)$ in Fig.1. All vehicles are moving along the specific closed curve in the phase space, which is a kind of limit cycle.

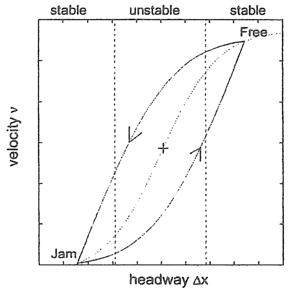

FIGURE 1. The profile of jam flow solution. The size of the loop depends on a, not on b.

Now, we first review the phenomena of synchronized flow. Figure 2 shows the temporal sequential data for the oscillation of average velocities in two parallel lanes at the three different points (0.56km, 2.3km and 4.7km) in congested flow on the upstream of the bottleneck [5]. The amplitude of oscillation is quite small at the immediate upstream of the bottleneck. The synchronization appears in the larger distance from the bottleneck. As the correlation between two lanes become clear, the amplitude of velocities become larger. Actually, the synchronized flow is followed by jam flow in much larger distance. So, the synchronized flow occurs in some intermediate distance from the bottleneck, between the small-amplitude flow and the jam flow.

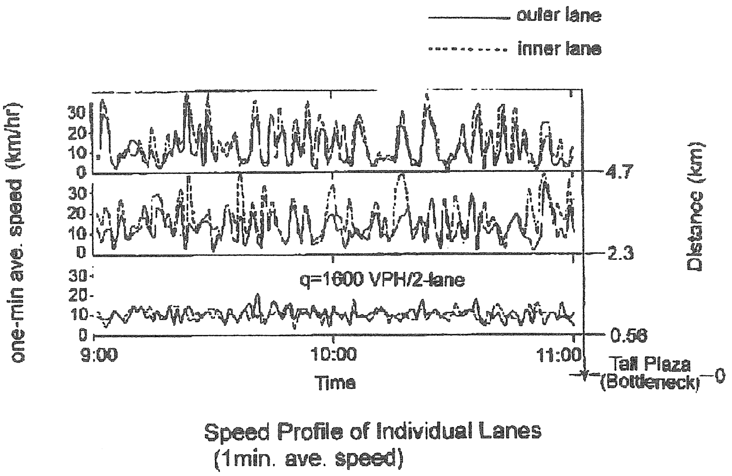

FIGURE 2. The temporal sequence of the velocity-changing of two lanes at three points upstream from the bottleneck.

We perform the simulations of OV model with a bottleneck in one-lane traffic of open boundary. The structure of flow on the upstream of the bottleneck, which consists of three distinct regions of flow mentioned above, can be observed even in one-lane.

We define the bottleneck using OV-function as

$$V_{bottle}(\Delta x) < V(\Delta x), \tag{3}$$

in the region of a bottleneck. This definition means the optimal velocity in a bottleneck (Here, we suppose a tunnel.) is reduced comparing with that outside it. The suppression rate of bottleneck is measured by the ratio of OV-functions as $S_{bottle} = V_{bottle}/V$.

We observe the formation of stable structure of traffic flow around the bottleneck. After relaxation, three distinct spatial temporal patterns of flow are formed on the upstream of the bottleneck as Fig.3. In the immediate upstream the vehicles are distributed almost uniformly. The fine stripes are observed in the further upstream region, which means the vehicle density is oscillatory changing with small period in this region. This flow of "oscillatory wave" is followed by the jam flow, where jam clusters appear clearly.

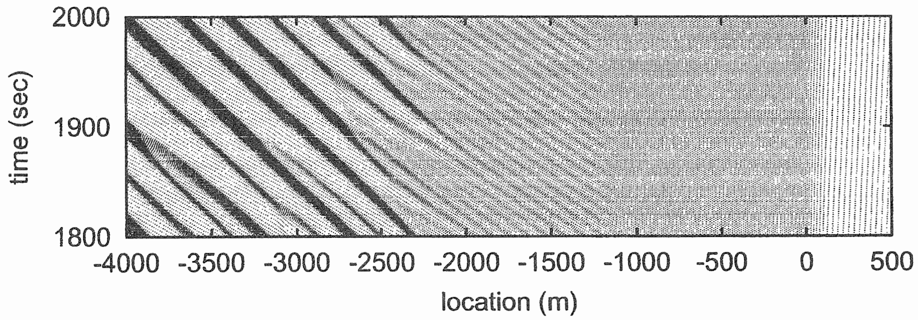

FIGURE 3. The three stable patterns on the upstream of the bottleneck for $S_{bottle} = 0.65$, $a = 1.6$ after optimization. The bottleneck is set at $0m$-point, in which length is $100m$.

Figure 4 shows the snapshot of velocity distributions corresponding to Fig.3. We can see the homogeneous flow with the identical velocity and headway in the region immediate upstream of the bottleneck. In the intermediate region 'the oscillatory wave' is observed for both velocity and headway. In the final region the kink-like cluster appears, which implies stop-and-go-wave as jam flow. Jam clusters propagates upstream, but the region of the oscillatory wave is located near but in some distance of the bottleneck.

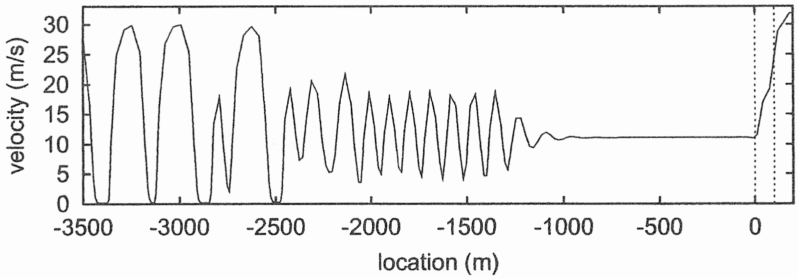

FIGURE 4. The snapshot of velocity distribution on the upstream of the bottleneck (the region between the dashed lines) of $S_{bottle} = 0.65$.

Figure 5 is the headway-velocity plots observed at four different distances from the bottleneck in Fig. 3. Each figure has the vehicle-points within $100m$ of the observation point in 5 minutes. Figure5(d) shows the transit-state from the homogeneous flow to the oscillatory wave. Figure5(c) shows the small closed loop as a clear trajectory in comparison with Fig.5(d). The small closed loop can be understood as the profile of the oscillatory wave. Within the interval from $-1300m$ to $-2000m$ the profile preserve its shape, that indicates the oscillatory wave is stable in this situation. This state is followed by the jam flow solution through the transit-state shown in Fig.5(b). The profile of jam flow in Fig.5(a) is just the same as that on the circuit with no bottleneck in Fig.1. The small closed loop in Fig.5(c) seems to be a kind of limit cycle solution as the jam flow solution is. This small limit cycle is not observed in the simulation on the circuit, that implies this solution is originally unstable but is convectively stabilized by the bottleneck. In this sense the small loop solution can be called 'quasi-stable' solution which can be stabilized in some boundary conditions such as a bottleneck.

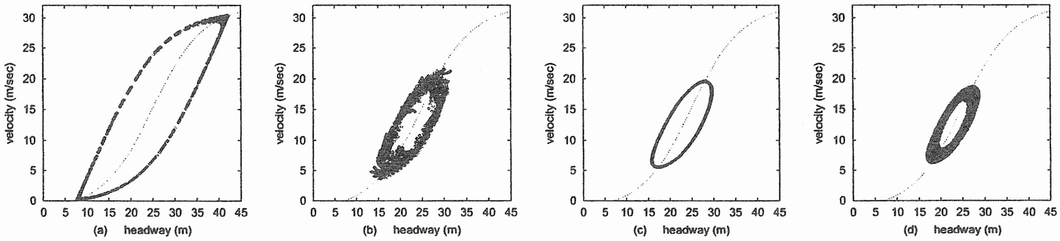

FIGURE 5. The plots of vehicles-points in the phase space (headway, velocity) at four points: (a) -3500 m, (b) -2300 m, (c) -1500 m, (d) -1200 m (from left to right), on the upstream from the bottleneck.

The flow of oscillatory wave has small amplitude comparing with jam flow and it is localized at some distance beyond the homogeneous flow upstream of the bottleneck. In flow-density plot, the average density varies with the same flow-rate. These properties indicate the possibility to understand the "synchronized flow" as the "oscillatory wave" solution of OV model with bottleneck. The similar oscillatory wave solution as ours, have been found by Mitarai and Nakanishi on the analysis of OV model in open boundary with the existence of localized perturbation [6]. They suggest the similar sequence of patterns on the flow upstream of the localized noise, which sustains the structure of their flow. Our investigation in this paper is for one-lane traffic, but the above properties must be essentially the same with two-lane traffic.

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