交通流の離散モデルの比較検討

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1 Introduction

The rule–184 CA has been widely used as a prototype of deterministic model of traffic flow. Several variations of it has been proposed recently: first, a high speed model of the rule–184 CA[1], i.e., cars can move more than one site per a time if there are vacant spaces ahead. Second, a "monitored traffic model"[2], which is considered as a kind of quick start(QS) model. Thus it moves *quickly* compared with the rule–184 CA. Third, a slow start(SlS) model[3], in which cars stop at a time cannot move at the next time and wait one time step to go forward. We call these variants as the rule–184 CA family in this paper.

Recently, a multi-value generalization of the rule–184 model has been proposed by using the ultradiscrete method[4]. The model is

$$U_j^{t+1} = U_j^t + \min(U_{j-1}^t, L - U_j^t) - \min(U_j^t, L - U_{j+1}^t), \tag{1}$$

where U_j^t represents the number of cars at site j at time t, and L is an integer. Each site are assumed to hold L cars at most. Since (1) is obtained from an ultradiscretization of the Burgers equation, and then we call it as the Burgers CA(BCA). It is noted the BCA contains the rule–184 CA as a special case of $L = 1$. We have introduced the positive integer L in (1), and physical meanings of it can be considered as following three ways: first, the road is L-lane freeway in a coarse sense, and effect of lane changes of cars is not considered explicitly. Second, we consider that U_j^t/L represents the probability of existing of a car at site j and time t in a single-lane freeway. In this case, the number U_j^t itself no longer represents real number of cars at site j. Third, also in a single-lane freeway we assume that the length of one site is long enough to contain the number of cars at most L. By introducing the free parameter L, the multi-value generalized CA has rich algebraic properties and wide applications to various transport phenomena. In this paper, we will generalize the whole CA's in the rule–184 CA family to multi-value ones. The multi-value CA models have rich properties, and we will study them by a *max*–puls representation throughout this paper.

2 Multi-value generalization

2.1 Multi-value SlS model

In the SlS model, standing cars cannot move soon at the next time, and they can move at two time steps later if there are vacant spaces in their next site. In the multi-value case, we should distinguish standing cars and moving cars in each site, and only standing cars need to wait one time step. The number of cars at the site $j-1$ blocked by cars in front of them at time $t-1$ is represented by $U_{j-1}^{t-1}-\min(U_{j-1}^{t-1}, L-U_j^{t-1})$. These cannot move at time t, then the maximum number of cars that move to the site j at t is given by $U_{j-1}^t - \{U_{j-1}^{t-1} - \min(U_{j-1}^{t-1}, L - U_j^{t-1})\}$. Therefore, considering the number of cars entering into and escaping from site j, the multi-value generalized SIS CA is given by

$$\begin{aligned} U_j^{t+1} = U_j^t &+ \min\left[U_{j-1}^t - \{U_{j-1}^{t-1} - \min(U_{j-1}^{t-1}, L - U_j^{t-1})\}, L - U_j^t\right] \\ &- \min\left[U_j^t - \{U_j^{t-1} - \min(U_j^{t-1}, L - U_{j+1}^{t-1})\}, L - U_{j+1}^t)\right] \end{aligned} \tag{2}$$

We note that this model includes the T^2 model in the case $L = 1$. The time neighborhood size is "3" in this model, and this represents an effect of inertia of cars.

2.2 EBCA1

Next, we extend the BCA to the velocity "2". We consider that cars moving one site have priority to those which can move two sites. This new model is called the EBCA1 in this paper.

Let us consider the evolutional rule of this model. One time step consists of following two successive procedures.

1) Cars move according to the BCA.

2) Only those cars that have moved in the procedure 1) can move more one site according to the BCA.

The number of cars at site $j$that can move forward in the first procedure is given by $b_j^t \equiv \min(U_j^t, L-U_{j+1}^t)$. In the next procedure 2), the number of cars that can move more one site is given by $\min(b_j^t, L - U_{j+2}^t - b_{j+1}^t + b_{j+2}^t)$, where the last term in min represents vacant spaces at site $j+2$ after the first procedure. Therefore, considering the number of cars entering into and escaping from site j, the evolutional rule of the EBCA1 is given by

$$\begin{aligned} U_j^{t+1} &= U_j^t + b_{j-1}^t - b_j^t \\ &\quad + \min(b_{j-2}^t, L - U_j^t - b_{j-1}^t + b_j^t) - \min(b_{j-1}^t, L - U_{j+1}^t - b_j^t + b_{j+1}^t) \\ &= U_j^t + \min(b_{j-1}^t + b_{j-2}^t, L - U_j^t + b_j^t) - \min(b_j^t + b_{j-1}^t, L - U_{j+1}^t + b_{j+1}^t). \end{aligned} \tag{3}$$

It is noted that (3) of $L = 1$ differs from the FI model. The EBCA1 becomes the rule-3372206272 CA in the case $L = 1$.

3 Fundamental diagram and multiple state

The fundamental diagram of new CA models discussed in the previous section is studied in detail in this section. In the followings, we will consider a periodic road, or a circuit. All models in Sec.2 are in a conserved form such as

$$\Delta_t U_j^t + \Delta_j q_j^t = 0, \tag{4}$$

where Δ_t and Δ_j are forward difference operator with respect to indicated variables, and q_j^t represents the traffic flow. The average density ρ^t and average flow Q^t over entire system is defined by

$$\rho^t \equiv \frac{1}{KL}\sum_{j=1}^{K} U_j^t, \tag{5}$$

$$Q^t \equiv \frac{1}{KL}\sum_{j=1}^{K} q_j^t. \tag{6}$$

where K is the number of sites in a period. Since in our models, the average density is a conserved quantity, then we will simply write it as ρ. Fig.1.1 and 1.2 are the density–flow diagram of each model with $L = 2$.

Fig.1.1

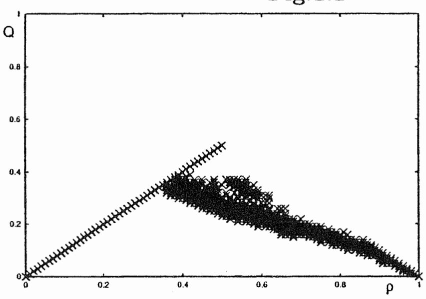
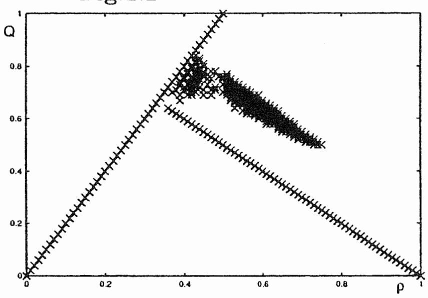
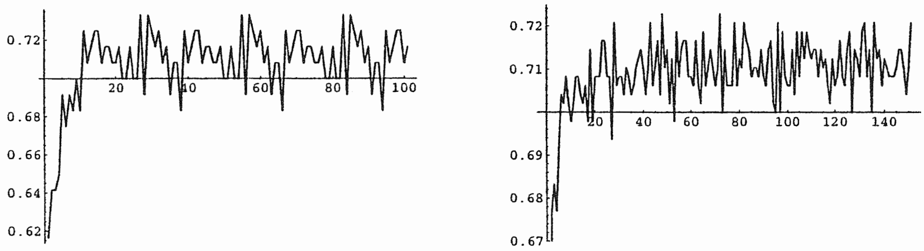


Fig.1.2



We plot the flow Q^t for $t = 2K$, which is sufficiently long enough to relax an initial configuration in the case of the BCA. We see that there are multiple states around the critical density, and a new small branch exists near critical density. It is interesting that from the fundamental diagrams the plotted data look like random, while the rules are completely deterministic. The fluctuation shows that the traffic flow near the transition region will never relax to a constant value in those models. We focus on the EBCA1 and let us examine the flow in detail. Fig.2 shows the time evolution of the flow Q^t in the EBCA1 starting from a state in the phase transition region.

Fig.2

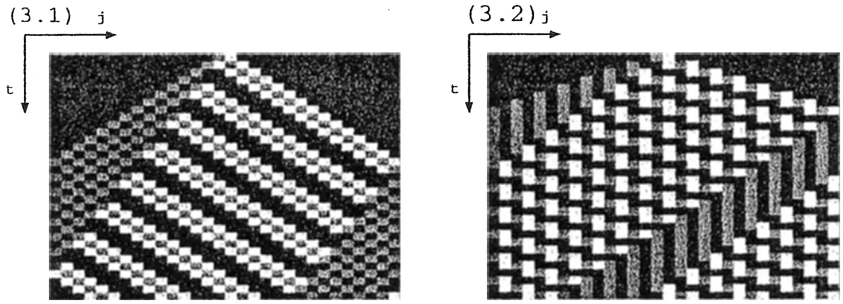


(2.1) K=60 (2.2) K=240

We can see that periodic irregular variation of flow appear after a short time. The power spectrum of it indicates that the irregular oscillation is white noise.

Finally, we study the stability of the uniform flow $\cdots 11111 \cdots$, which has the maximum flow in each model. We define a perturbation that changes the state $\cdots 11 \cdots$ to $\cdots 20 \cdots$. Fig.3.1 and 3.2 show an instability of uniform flow by the perturbation of SIS model and EBCA1, respectively.

Fig.3



By the weak perturbation, the flow decreases and transits to the lower flow state.

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