CHAOS CONTROL IN SOME TRAFFIC FLOW MODELS

Elman Mohammed Shahverdiev,[1]
Department of Information Science,[2] Saga University, Saga 840, Japan
Shin-ichi Tadaki,[3]
Department of Information Science, Saga University, Saga 840, Japan

ABSTRACT

Chaos control in some of the one- and two-dimensional traffic flow dynamical models in the mean field theory is studied.One dimensional model is investigated taking into account the effect of random delay. Two dimensional model takes into account the effects of overpasses, symmetric distribution of cars and blockages of cars moving in the same direction. Chaos control is performed within both replica and nonreplica approaches, and using parameter perturbation method.

Nowadays traffic flow problems has acquired interdisciplinary status.see, e.g. refs.1-10.Mean field theory and cellular automaton (CA) models have extensive applications to the traffic flow problems. In this paper we deal with two models in traffic flow dynamics in the mean field theory:one- and two-dimensional systems in the context of chaos control theory possibilities in traffic problems. The outline of this paper is the following:first we present both one- and two-dimensional traffic models in question; then we apply some chaos control methods to these dynamical systems.We begin with one dimensional model.

Recently, the authors of ref.9 have presented microscopic derivations of mean field theories for CA models of traffic flow in one dimension.They established the following mapping between the average velocities of cars v at times t+1 and t:

$$v(t+1) = (2-f)v(t) - (1-p)^{-1}v^2(t) + p(1-p)^{-1}v^3(t), \quad (1)$$

where,p is the car density; f is the quantity responsible for the random delay due to the, say, different driving habits and road conditions. So one has discrete dynamical nonlinear system, which exhibits rich dynamical behavior.

Not so long ago Biham et al.(ref.2) (below simply BML) introduced a simple two dimensional (2D) CA model with traffic lights and studied the average velocity of cars as a function of their density.In that model, cars moving from west to east attempt to move in odd time steps and cars from south to north -in even time steps.There are three possible states on the square lattice:(i) occupied by an eastbound car; (ii)occupied by a northbound car; (iii)vacant. In ref.8 it was underlined that the divison of time into odd and even time steps simulates the effect of of traffic lights.The main result of BML:the average velocity in the long time limit vanishes when the density of cars p is higher than a critical value p_c.Below

[1]e-mail:shahverdiev@lan.ab.az
On leave from Institute of Physics, 33, H.Javid avenue, Baku 370143, Azerbaijan
[2]e-mail:elman@ai.is.saga-u.ac.jp
[3]e-mail:tadaki@ai.is.saga-u.ac.jp

p_c, the traffic is in a moving phase, while above p_c it is in a jamming phase.Numerous improvements of the basic BML model taking into account the effects of factors such as overpasses, faulty traffic lights, asymmetric distribution of cars in a homogenous lattice, and traffic accidents,(see ref.8 and refenences therein.)has been carried out. In the above-cited recent paper (ref.8) an improved mean field theory for 2D traffic flow with a fraction c of overpass sites and with possible asymmetry in the distributions of cars in the two different directions is studied. The model in fact is the improved version of Nagatani model, ref.11. The overpass sites can be occupied simultaneously by an eastbound car and a northbound car, thus modelling the two-level overpasses in modern road systems in cities.Nagatani model deals with the isotropic distribution of cars with different overpass sites.It has been shown that the addition of overpasses enhances both the average speed of the traffic and critical density of cars.However the Nagatani model has some shortcomings in the sense that blockage of cars due to cars moving in the same direction is not taken into account properly, which led to too low estimation of the concentration of overpasses for the transition from jamming to moving phase at $p = 1$ and too high estimate of the critical car density at $c = 0$. Although in this paper we will deal with the isotropic distributions of cars for the sake of completeness we first write the system of equations with asymmetry. Let p_x and p_y to be the density of cars in the x (eastbound) and y(northbound) directions respectively.Also let v_x and v_y to be the average speeds of cars in the same directions;Let also c to be a fraction of overpass sites.Then according to ref.8 these quantities are related through the following nonlinear dynamical equations:

$$v_x = 1 - (1-c)(p_y v_y^{-1} + p_x(v_x^{-1} - 1))$$

$$v_y = 1 - (1-c)(p_x v_x^{-1} + p_y(v_y^{-1} - 1)), \qquad (2)$$

From the matematical point of view the system (2)also is the nonlinear dynamical system.
It is well-known that some dynamical systems depending on the value of systems' parameters exhibit unpredictable,chaotic behaviour,refs.10-15. The seminal papers (refs.12-13) induced avalanche of research works in the theory of control of chaos in synergetics.Chaos synchronization in dynamical systems is one of such ways of controlling chaos. In the spirit of refs.12-13 by synchronization of two systems we mean that the trajectories of one of the systems will converge to the same values as the other and they will remain in step with each other. For the chaotic systems synchronization is performed by the linking of chaotic systems with a common signal or signals (the so-called drivers)
According to refs.12-13 in the above mentioned way of chaos control one or some of these state variables can be used as an input to drive a subsystem that is a replica of part of the original system.In refs.12-13 it has been shown that if all the Lyapunov exponents (or the largest Lyapunov exponent) or the real parts of these exponents for the subsystem are negative then the subsystem synchronizes to the chaotic evolution of original system.If the largest subsystem Lyapunov exponent is not negative then as it has been proved in ref.18 synchronism is also possible. In this case a nonreplica system constructed according to ref.18 is used instead of replica subsystem.
The interest to the chaos synchronization in part is due to the application of this phenomenen in secure communications, in modeling of brain activity and recognition processes, etc, see,references in ref. 17).

Also we should mention that this method of chaos control may result in the improved performance (according to some criterion) of chaotic systems (see, e.g.ref.17).
In this paper for the first time (to our best knowledge) we report on the possible chaos control in the traffic flow models.
Namely our paper is dedicated to the study of the stablization of unstable behaviors in one- and two-dimensional traffic models with the proportional feedback, replica and nonreplica and parameter change methods in chaos control theory.
First we investigate the one dimensional model.The stationary values of average velocity can be easily calculated from the equation (1).These values are:

$$v_1^{st} = 0, v_2^{st} = \frac{1-(1-4(1-f)p(1-p))^{\frac{1}{2}}}{2p}, \qquad (3)$$

The stability analysis of this stationary states show that :the v_1^{st} is always unstable,except for $p = 1$. Indeed,for this state

$$|\frac{v(t+1)}{v(t)}| = |(2-f)| > 1, \qquad (4)$$

As f changes between zero and unity.The instability condition for the second stationary state is:

$$|(1-p)^{-1}\frac{3(1-4(1-f)p(1-p))-1-2(1-4(1-f)p(1-p))^{\frac{1}{2}}}{4p} + 2 - f| > 1, \quad (5)$$

As the stability analysis show in general we have stable and unstable states depending on the value of p, f. As a rule the unstable states are discarded as unphysical ones.But nowadays due to the success of chaos control theory it is possible to stabilize the unstable fixed points or periodic orbits. Below in dealing with the control of instability of fixed points in one dimensional map we will follow the so-called proportional feedback method described in ref.19, which is map based variation of a method proposed in ref.12.Following the method presented in ref.19 first we linearize the one dimensional map (1) in the vicinity of the fixed points (or stationary states v^{st}):

$$v(t+1) = h(v(t) - v^{st}) + v^{st}, \qquad (6)$$

where $|h| > 1$ is the slope of the map at v^{st}. In the mapping (1) we have only one parameter f by changing which one can stabilize the unstable fixed points.The positive answer to this problem vindicates the intuition that by improving drving habit,road conditions one cansolve some traffic difficulties.Of course, theoretically the regularization of traffic flow also could be achieved by manipulation with p, say by the regulation of cars entrance to the traffic flow. Having this in mind let us denote the parameters as m=(f,p).Now suppose that we change this parameter m by small amount δm to move the unstable fixed point v^{st} without significant changing of the slope of the map (1) h. In other words

$$v_{t+1}(m + \delta m) = h(v_t - v^{st}(m + \delta m)) + v^{st}(m + \delta m), \qquad (7)$$

(in order to avoid confusion, in some formulai we write t as a subscript) where

$$v^{st}(m+\delta m) = \delta m \frac{dv^{st}}{dm} + v^{st}, \qquad (8)$$

Now suppose that$v_t = v^{st}(m) + \delta_1 v$,where the second term in the right-hand side of this equality is much smaller than the first one. If at this moment m is changed to $m + \delta m$ such that $v_{t+1}(m+\delta m) = v^{st}(m)$, the system state is directed to the original unstable fixed point upon the next iteration.If m is then switched back to its original value, the system would remain at v^{st} indefinitely. The necessary variations of m can easily be determined by the formula

$$\delta m = \frac{h}{(h-1)\frac{dv^{st}}{dm}} \delta_1 v = \frac{\delta_1 v}{g}, \quad (9)$$

One can see easily the necessary changing of parameters to stabilise the unstable fixed points is proportional to the deviation of v from the fixed point (or stationary state).That is why the method is called the proportional -feedback one. So the use of the proportional feedback method allows one to stabilize unstable stationary states. Speaking about possible ways of chaos control in one dimensional systems in the context of traffic flow instabilites one should also mention about such a control by changing of dynamical variable (velocities of cars) itself, ref.20.In this connection it is worth while to mention the refs.10,21 where the velocity of cars could become much larger than unity.

Now we study the two dimensional case.Below, as mentioned above we restrict ourselves to the isotropic case.

The system of equation (2)can be regarded as a mapping describing the time evolution of the velocity in the moving phase with $v_x(t+1)$ and $v_y(t+1)$ on the left-hand side and $v_x(t)$, $v_y(t)$ on the right-hand side.

In the case of isotropic distribution this mapping can be written as:

$$v_x(t+1) = 1 - (1-c)(\frac{p}{2}v_y^{-1} + \frac{p}{2}(v_x^{-1} - 1)) = F_1,$$

$$v_y(t+1) = 1 - (1-c)(\frac{p}{2}v_x^{-1} + \frac{p}{2}(v_y^{-1} - 1)) = F_2, \qquad (10)$$

where $p_x = p_y = \frac{p}{2}$.

The system of nonlinear mapping has two steady state solutions

$$v_{\pm} = \frac{1}{2}(1 + \frac{(1-c)p}{2} \pm ((1 + \frac{(1-c)p}{2})^2 - 4(1-c)p)^{\frac{1}{2}}), \qquad (11)$$

where $v = v_x = v_y$.

First of all we should find the condition of possible chaoticity in the system (10).

The stability of mapping is determined by the eigenvalues of the Jacobian matrix of the nonlinear mapping (10).

$$J = \frac{\partial(F_1, F_2)}{\partial(v_x(t), v_y(t))}, \qquad (12)$$

It can be seen easily the eigenvalues of the Jacobian matrix is calculated by the following equation:

$$\lambda^2 - \lambda(1-c)\frac{p}{v^2} = 0, \qquad (13)$$

From here we obtain easily that

$$\lambda_1 = 0,$$

$$\lambda_2 = (1-c)\frac{p}{v^2}, \qquad (14)$$

In the last expression while calculating λ we use the steady state solutions (10). This simplification is justified at least for systems whose chaotic behavior has arisen out of stability of fixed points, see ref.15, also ref.22-23.

The mapping will exhibit chaotic behaviour, if the absolute values of λ exceed unity.

$$|\lambda_2| > 1, \qquad (15)$$

. One can see easily if the car density exceeds the critical one (see,ref.8) then jamming is taking place, and the average velocity will be diminished thus promoting the congestion and chaoticity. As the initial or original nonlinear mapping is symmetric over v_x and v_y considering only one of these variables as a driver will be sufficient.Take for the definiteness v_x variable as a driver. Then the replica subsystem(with the superscript"r") can be written as follows:

$$v_y^r(t+1) = 1 - (1-c)(\frac{p}{2v_x(t)} + \frac{p}{2}(\frac{1}{v_y^r(t)} - 1)) = H, \qquad (16)$$

Then the Lyapunov exponent can be calculated as follows,ref.18

$$\Lambda = \ln\frac{\partial H}{\partial v_y^r} = \ln(1-c)\frac{p}{2}\frac{1}{v^2}, \qquad (17)$$

. For the chaos control (to be more specific for the synchronization of the evolution of the response system to the chaotic evolution of the initial nonlinear mapping when time goes to infinity) it is required that

$$\Lambda < 0, \qquad (18)$$

. It will take place, if the following condition is satisfied:

$$(1-c)\frac{p}{2v^2} < 1, \qquad (19)$$

. Thus we have two inequalities:(19) and (15).If these inequalities do not contradict each other, then the replica approach allows us to perform chaos control. We see that chaos control within replica approach is realizable if

$$1 < (1-c)p\frac{1}{v^2} < 2, \qquad (20)$$

Although the restriction (20) is very severe in the sense that diapason of changing of traffic flow models parameters such as c, p could be very narrow.Nevertheless, as the analysis of the data presented in ref.8 shows that chaos synchronization would be possible within the replica approach.If this approach fails, we can apply nonreplica one to achieve our goal,as it was underlined above.
According to ref.18, within nonreplica approach we can use the following nonreplica response system(with the superscript " nr"):

$$v_x^{nr}(t+1) = 1-(1-c)(\frac{p}{2}(v_y^{nr})^{-1} + \frac{p}{2}(v_x^{-1}-1)) + \alpha(v_x^{nr}-v_x) = F_3,$$

$$v_y^{nr}(t+1) = 1-(1-c)(\frac{p}{2}v_x^{-1} + \frac{p}{2}((v_y^{nr})^{-1}-1)) + \beta(v_x^{nr}-v_x) = F_4, \qquad (21)$$

where α and β are the arbitrary constants.
Here again as in the previous case we consider v_x dynamical variable as the driver.
The Lyapunov exponents are the eigenvalues of the Jacobian:

$$J = \frac{\partial(F_3, F_4)}{\partial(v_x^{nr}(t), v_y^{nr}(t))}, \qquad (22)$$

From (21) we easily establish that these exponents are solutions to the following equation:

$$\lambda^2 - \lambda(\alpha + (1-c)\frac{p}{2}v^{-2}) - \beta(1-c)\frac{p}{2}v^{-2} = 0, \qquad (23)$$

Here v is the steady state solution of the original mapping. As it can be seen from (23) the roots of this equation λ_1 and λ_2 satisfy the relationships:

$$\lambda_1 + \lambda_2 = \alpha + (1-c)\frac{p}{2}v^{-2},$$

$$\lambda_1\lambda_2 = -\beta(1-c)\frac{p}{2}v^{-2}, \qquad (24)$$

Remind that our aim is to satisfy the conditions $|\lambda_1| < 1$ and $|\lambda_2| < 1$. Due to the arbitraryness of the constants α and β this can be done easily.
Up to now while performing chaos synchronization we have taken the advantage of using the presence of driving variables explicitly.Our calculations show that chaos synchronization is also reachable in case of parameter perturbation method,ref.24. Namely,we show that by changing the fraction of overpasses one can make the absolute values of $\lambda_{1,2}$ less than unity. (In ref.24 this theory was proposed for continuous

dynamical systems.Here we readapt that approach for the case of discrete dynamical systems.). Indeed. by assuming the following change for c :

$$c = c_1 - \alpha_c(v_y - v_{y_{ap}}), \qquad (25)$$

(where:c_1 is the nominal value for the c in the original two dimensional model;α_c is the control coefficient to be found;v_y and $v_{y_{ap}}$ are are response system and drive system orbits,respectively.) after simple calculations we obtain that for the isotropic case much sought eigenvalues are:

$$\lambda_1 = 0,$$

$$\lambda_2 = 2(1-c_1)\frac{p}{2}v_{ap}^{-2} + \frac{p}{2}\alpha_c(1-2v_{ap}^{-1}), \qquad (26)$$

Thus we have the real possibility to satisfy the condition for the chaos control:$|\lambda_2| < 1$.

In conclusion in this work we pointed out to the possibility of the stabilization of the unstable stationary states in one of one dimensional model with random delay with chaos control methods. Also we have investigated the possibilty of chaos control in one of two dimensional mapping in traffic flow within replica, nonreplica and parameter change approaches.We showed that there is a real possibility of using chaos control methods in traffic flow problems.(Here in this paper we just briefly underlined some of chaos control methods in connection with traffic flow control problems.More detailed and extensive results will be published elsewhere.).

Speaking about the study of 2D CA traffic flow models one has to mention that although 2D CA models less representative (in comparison with 1D rule-184 CA) of real traffic flow, however they may be applicable to abstract traffic problems such as data packets in computer networks,ref.25. Besides, 2D CA models may be useful from the viewpoint of complex behavior in deterministic dynamics, ref.26-27.

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